PAPER MATH

Interesting Paper Activities all with an A4 Sized Sheet!

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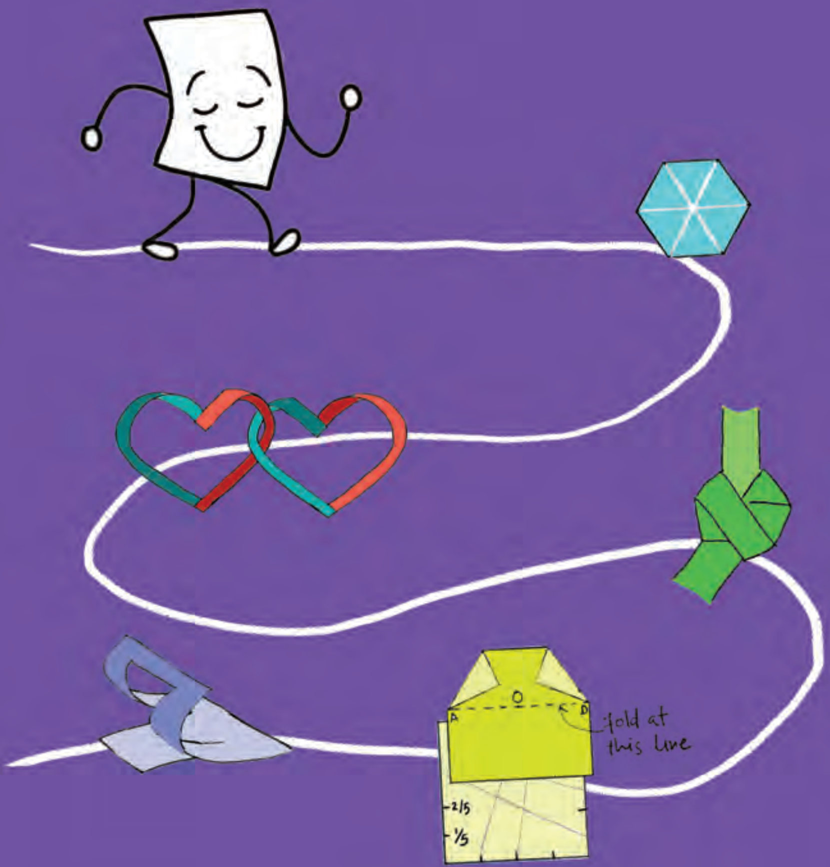

CENTER FOR CREATIVE LEARNING
IIT GANDHINAGAR

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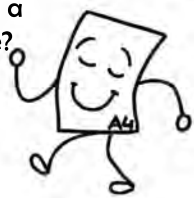
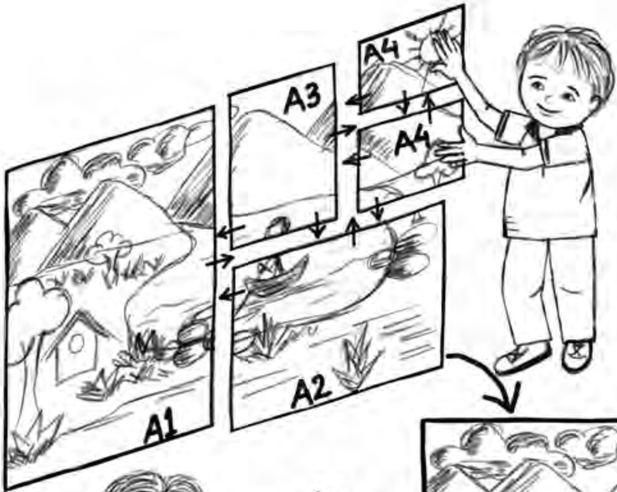
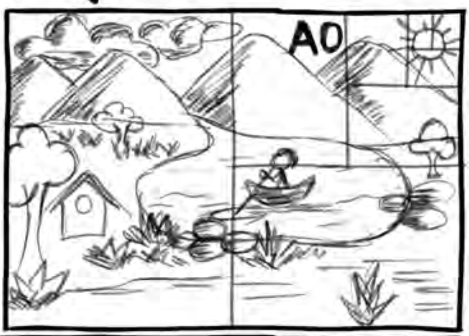
1 A4 Sheet Ratio of Sides

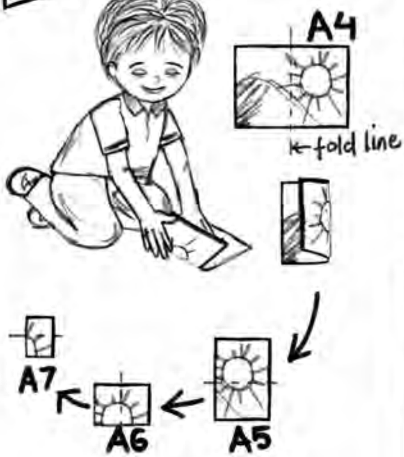
We see and use A4 sheets almost daily. We all know that it is a rectangle. But is there something special about this rectangle?

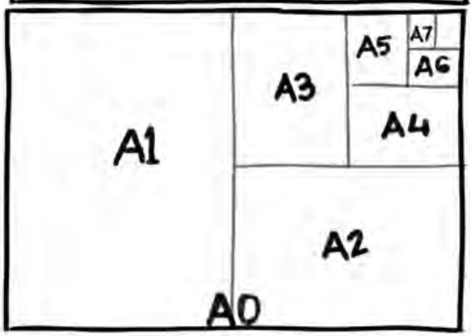




When we keep two A4 sheets side by side, We get an A3 sheet. With two A3 sheets, we get A2 and so on.

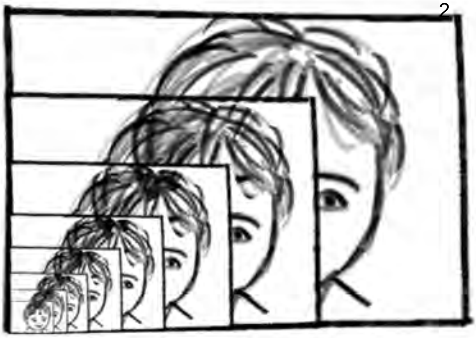






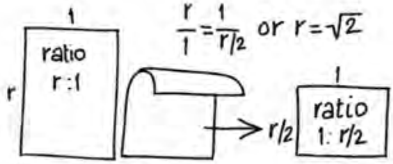
Similarly, if we go in other direction and halve the A4 sheet, we get an A5 sheet.

The speciality of this A-series is that the ratio or proportion of length and breadtth of all the sheets (A0, A1, A2, A3, A4,...) is exactly the same.



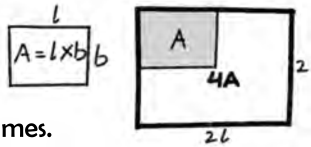
This same ratio implies that the photo or the text to be printed on the paper doesn't get stretched or squeezed, no matter the paper we choose for printing (A3, A4, A5,....). So if we want to change the type of paper, we don't have to adjust the aspect ratio. It is as if the same sheet has been zoomed!

Using this information, let's find this ratio.
Let's assume that the initial ratio of length to breadth is r:1. When the length is halved by folding the length, the longer side becomes 1 and the smaller side is r/2.
So the ratio is $\frac{1}{(r/2)}$.
And we are saying that these two ratios are equal.



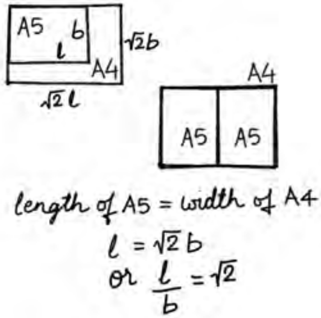
So $\frac{r}{1} = \frac{1}{(r/2)}$
Solving it gives, $r = \sqrt{2}$

We can also find the ratio without using the equations.
Area of a rectangular sheet = length x breadth
If we double it's dimensions the area becomes 4 times.



When we join two A5 sheets we get an A4 sheet. And the area becomes double in the process. And to increase the area by 2 times, we have to increase the length and breadth by $\sqrt{2}$ or 1.414 times (1.414x1.414 =2). Actually this is the definition of $\sqrt{2}$, the number which multiplied by itself gives the number 2.

For example, the area of A4 sheet is double that of A5. Therefore, the length and breadth of an A4 sheet is $\sqrt{2}$ times the length and breadth of an A5 sheet respectively. And if we look closely, the breadth of the A4 sheet is the length of the A5 sheet. So when we say that the ratio of length of A4 to length of A5 is $\sqrt{2}$, it also means that the ratio of length and breadth of A4 sheet is also $\sqrt{2}$.



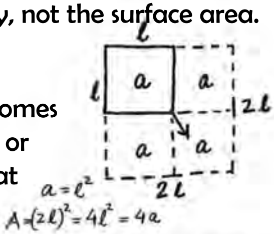
2 Who Feels Colder, Baby or Adult?

If we have two identical persons, one the size of a baby and the other one the size of an adult, which one would feel more cold- the baby or the adult?

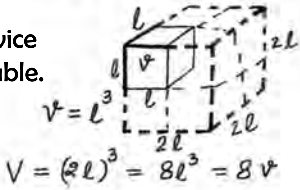
Let's find out. We feel cold when the skin is warmer than the surroundings. And the more skin we have in contact with the surroundings, the colder we feel. So it seems that the baby will feel less cold because the area of his skin is lesser than the adult. Right? Not quite.

DO you Know??? The size of cells of an adult and a child is same! So when we grow up, the number of cells in our body increases but the size of individual cells remains same.

We have to take into account one more process- the production of heat. A biger body is made up of more cells,/and these cells will generate more heat. As the cell are present in our whole body and not just in the skin(or the surface), the amount of cells is proportional to the volume of the body, not the surface area.



When all the sides of an object are doubled, the area becomes 4 times. And this is true for any shape, not just the square or the rectangle. This means that the skin through which heat loss takes place becomes 4 times.



Let's asuume that all the dimensions of the adult are twice of the baby. The length, breadth and height all are double.

Now let's see what happens to the volume when the dimensions are doubled. The volume becomes 8 times in this process, and with that, the ability to generate heat also becomes 8 times.

So although an adult loses more heat to the surrounding, his heat-generation capacity has also increased. The increase in heat production is more than the increase in heat-loss. That's why a child is dressed more heavily.

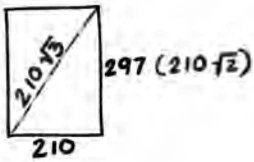

a = 6X(1)^2 = 6
v = (1)^3 = 1
double all lengths
A = (2)^2 X6 = 24 = 4a
V = (2)^3 = 8 = 8v

3 Square Roots in A4

A4 sheets are extremely special. Or for that matter any series of A paper or B paper or C paper. As we saw earlier that the sides are in ratio of √2.

What this means is that we can see three basic square roots in any sheet. For instance, an A4 sheet has the dimensions 297x 210 mm.

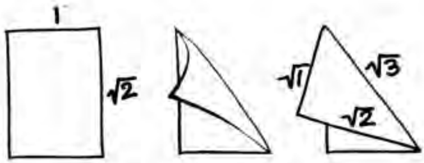
The ratio of the sides: 297/210 = √2

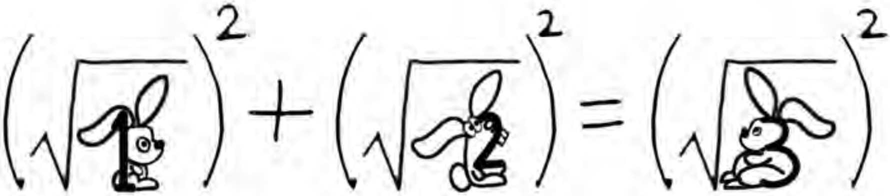


If we assume the width of A4 sheet to be 1 unit, the length is √2 which we have just shown. Now if we fold the paper along the diagonal, and find the length of this diagonal using pythagoras rule, it would be

$$\sqrt{(1^2 + (\sqrt{2})^2)}$$

which comes out to be √3.

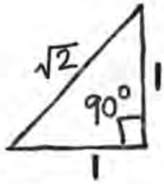




So the first three square roots are there in the most-used sheet of paper.
How interesting!

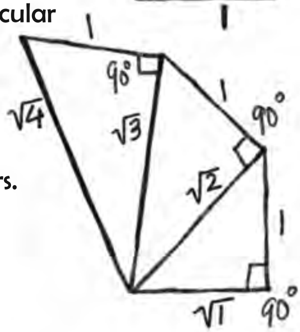
4 Root Spiral
√1, √2, √3,..

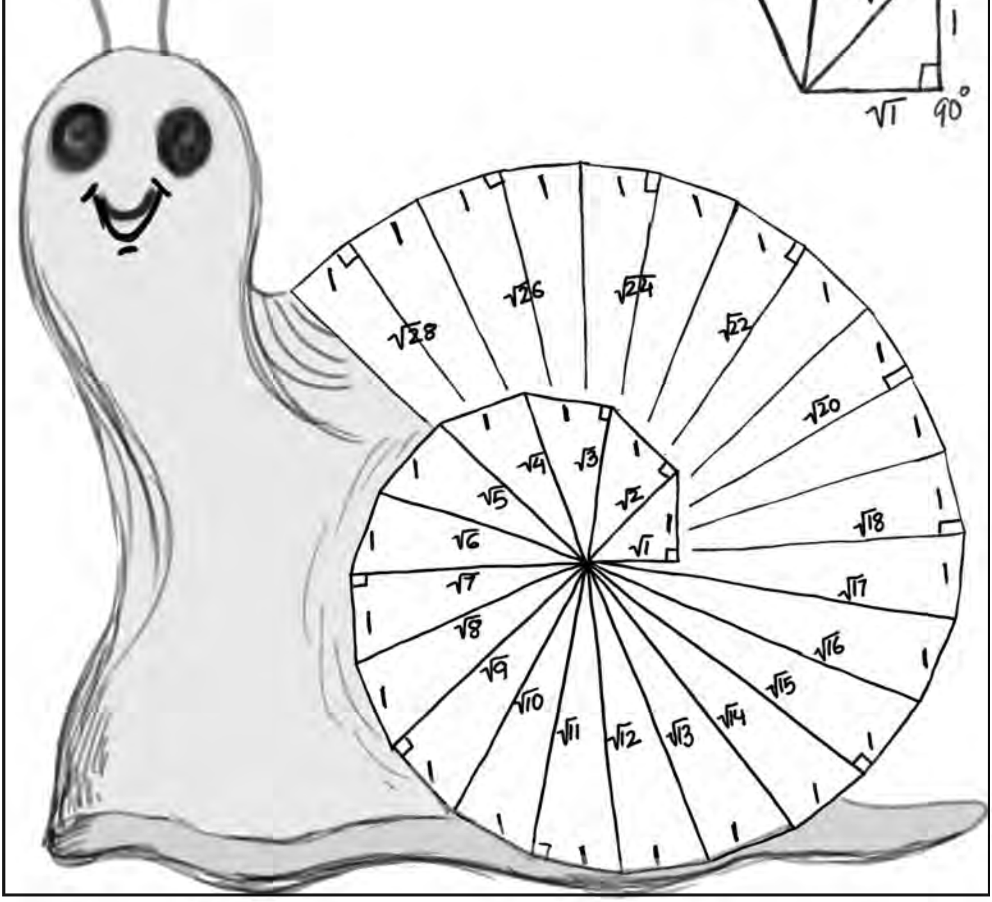
We can construct a spiral of square roots, starting with an isosceles right triangle of unit length. If the perpendicular and base are of unit length, the length of hypotenuse comes out to be $\sqrt{(1^2+1^2)} = \sqrt{2}$. (by pythagoras theorem)



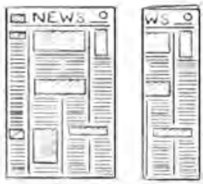
Now if we draw another line of unit length perpendicular to this hypotenuse, length of the new hypotenuse obtained is $\sqrt{(1^2 + (\sqrt{2})^2)}$.

We can continue like this and get sides of length equal to the square roots of subsequent numbers.



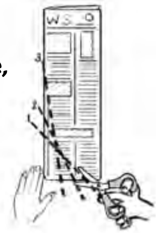


5 Isosceles Triangle from a Newspaper



Take a newspaper and fold it into half along the length.

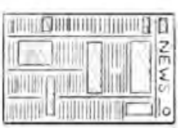
Cut a piece of paper from the folded side at any angle, but in a straight line.

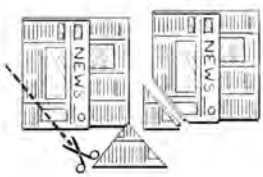


Unfold this piece...you get an isosceles triangle.

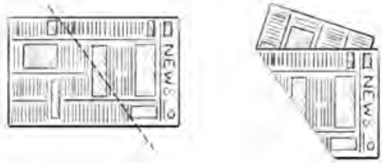
Why did you get so? Because when we folded the paper it gave us the perpendicular bisector for the triangle. As we had cut the two sides of the triangle by the same cut we get the sides equal.

Now, what will happen if we fold it along the width and also not even from the middle? The result will be same, because we had folded paper perpendicularly only.

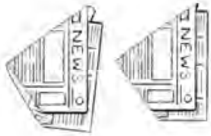




What will happen if we fold the paper obliquely?



We can still get an isosceles triangle, but we will have to make one more fold, folding the folded edge exactly upon itself.



Cut the paper on this fold.

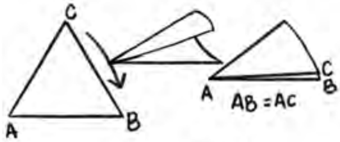
Now cut a piece of paper from the folded side by making an oblique cut.

We get an isosceles triangle.

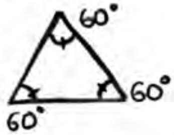
6 Equilateral Triangle from A4 Sheet

An equilateral triangle is a triangle in which all three sides are equal. We will try to make an equilateral triangle by folding an A4 sheet and that too without using a scale or compass. Before going any further, try it yourself and fold an A4 sheet to make a triangle.

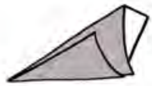
How can we ensure that the triangle we get after folding the sheet is an equilateral triangle? Just put one side on top of another to check whether they are equal.



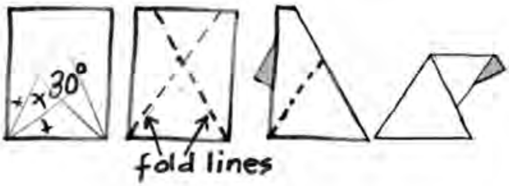
One of the ways is to use the fact that all angles of an equilateral triangle are 60^{o}. We can fold angles of 60^{o} by dividing the angle of 90^{o} in 3 equal parts and then taking two of those parts.

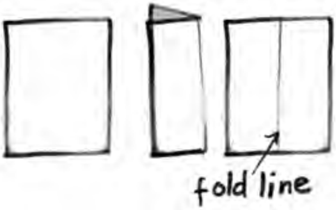


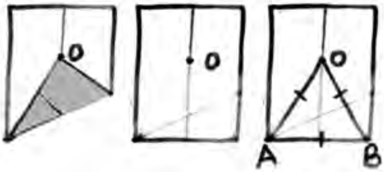
But how do we divide an angle in three equal parts? Just fold the angle such that the remaining part of the angle completely covers the initial fold. Doing this for two corners of A4 sheet gives us the equilateral triangle.



But this method of trisecting the angle is approximate because we can't exactly divide an angle in three equal parts the process of trisecting an angle always involve some trial and error.





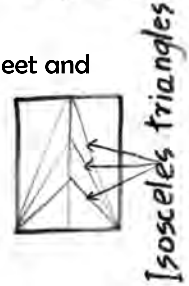


One more way is to first fold the width of the paper in half. The crease that we get would divide it in two equal parts.

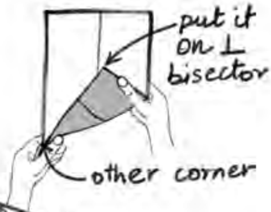
Now lift the width of the paper and put one of its corners on the crease as shown in the image. This is the third vertex of the triangle and the triangle we've got is surely an equilateral triangle.
Can you prove why?

Let's see. We will use an interesting property of perpendicular bisector that all the points on it are equidistant from the corners. And the first crease we got by folding the width in half was indeed the perpendicular bisector of the width.

All the triangles we get with these 3 points- 2 corners of the sheet and 1 point on the perpendicular bisector are at least isosceles. Now all we have to do is find a point on the perpendicular bisector line so that thel two lines drawn from it are equal to the base. Isosceles triangle would then become equilateral.



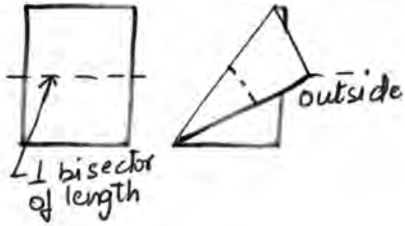
To do this, we lift the width of the paper and put it on the perpendicular bisector such that the fold line passes through the other corner. This ensures that the two lines we draw from vertex on the perpendicular bisector are equal to the base. Now we have made sure that all the sides are equal in length. Hence, our triangle is an equilateral triangle.



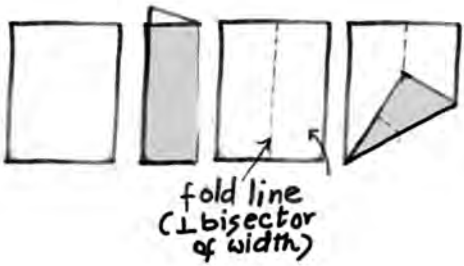
7 Largest Equilateral Triangle from A4 Sheet

What is the largest equilateral triangle we can make from an A4 sheet?

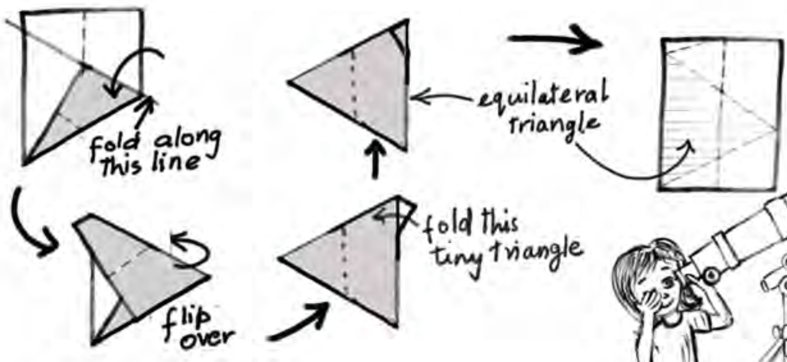
In the previous activity, we made one with sides equal to the width of A4 sheet. Now try making one with sides equal to the *length* of A4 sheet. You won't be able to do it because the third vertex will go out of the paper.



To fold the largest triangle, fold the perpendicular bisector of the width. Then lift the width of the paper and put it on the perpendicular bisector, such that the fold line passes through the other corner.
We have followed the steps of the previous activity till now.



Now fold the remaining paper as shown below.

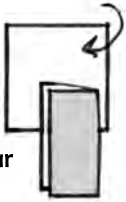


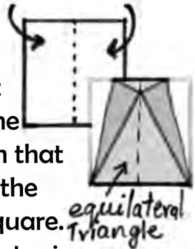
and we will get the largest equilateral triangle possible in an A4 sheet.

8 Largest Equilateral Triangle from a Square

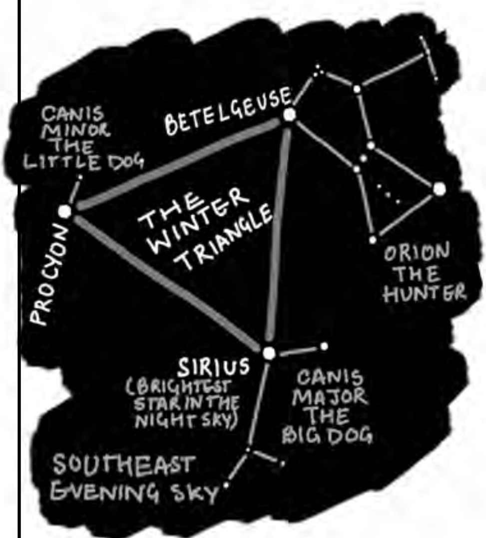
Making an equilateral triangle from a square is simpler because we already have 4 equal sides. The triangle formed by taking any three sides of the square will surely be equilateral.

To do this, first fold the square from the middle such that we get the perpendicular bisector of the side.



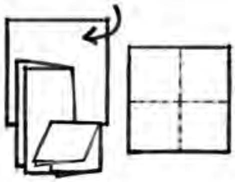


Now put the two adjacent vertices of the square on the perpendicular bisector such that the fold lines pass through the other two vertices of the square. Now the triangle at the center is an equilateral triangle because it is formed by taking 3 sides of the square which are equal.

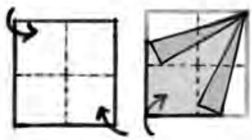


One more way is to proceed exactly like we did for an A4 sheet.

For making largest equilateral triangle from a square, fold the sides of square in half which would divide the square in four equal parts.

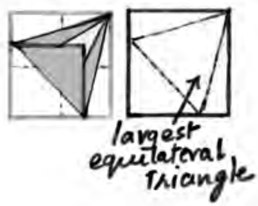


Then lift the side of square and put the corner on the crease as shown.



Now take another side perpendicular to the side taken in the previous step and put this on the second crease as shown.

Fold the remaining paper and the triangle obtained like this is the largest equilateral triangle in a square and has sides bigger than the square and has sides bigger than the square.



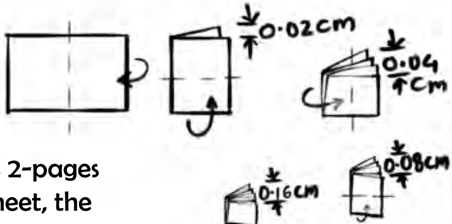
If we start with a unit square, sides of this largest triangle are 1.03527 units.

9 Going to the moon

How many times can we fold a sheet of paper onto itself?
Take an A4 sheet and try for yourself. You would probably be able to fold it six or seven times. But what if we take a really huge paper and keep folding it. Can we reach the moon by folding this paper onto itself?

We don't even know the thickness of a single sheet of paper so let's find out that first. Well, it's difficult to guess but we have seen a ream of 500 sheets which is around 5 cm thick. So the thickness of a single sheet is around 5/500 = 0.01 cm.

Now the distance of the moon from the earth is 3,84,400 km or
(3,84,400 km/0.01 cm)= 3,844,000,000,000 pages.
So it seems that we would need a lot of folds to go to the moon.
Let's find out.



Now if we fold this sheet once, it becomes 2-pages thick- 0.02 cm. Now, when we fold this sheet, the thickness becomes 0.04 cm. 3rd time, 0.08 cm. 4th time, 0.16 cm...10th time, 10.2 cm. Before going further, pick a number which you think is the closest to the number of folds required to go to the moon.

A. 40 B) 400 C) 4,000 D) 40,000

For every fold, the thickness is doubling. The increase in thickness is exponential and not linear. So rate of increase of thickness also picks up pace after some time. We can also write a formula for the thickness after n folds: 0.01×2^n

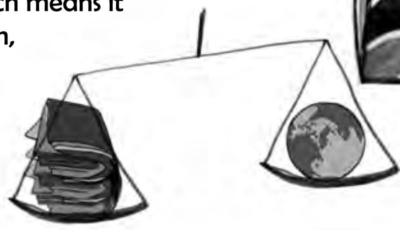
In 27 folds, the stack becomes 13.4 km high, higher than the Mt. Everest.

In 41 folds,
we would be halfway to the moon.

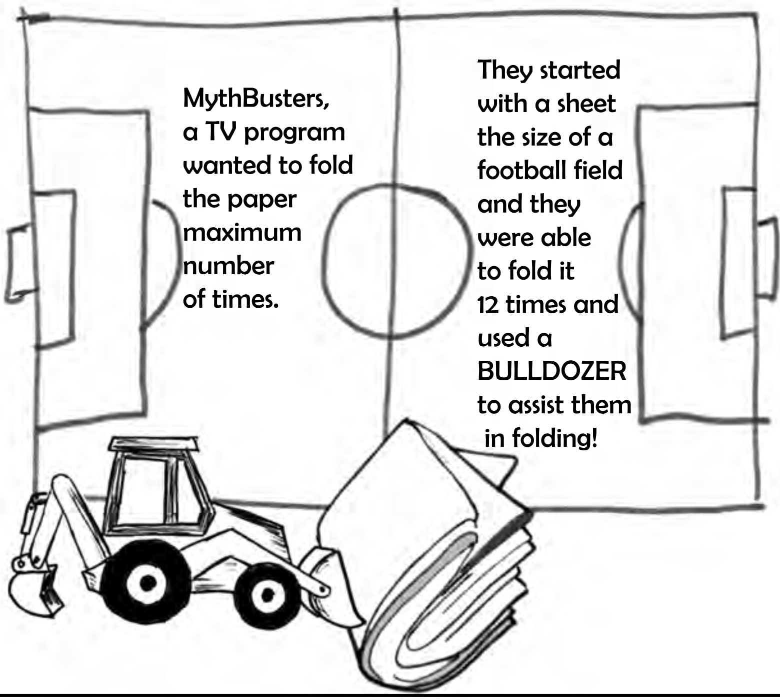
And all the way to the moon in 42 folds!

Of course there would be some logistical issues. As the area of the sheet is also decreasing exponentially, getting halved in every fold, you'd probably need to start with a sheet roughly the size of the milky way.

And at that point, the paper would most likely have more mass than the Earth, which means it would be compressing the Earth, not the other way around.



And good luck folding the paper more than 8-10 times in reality.



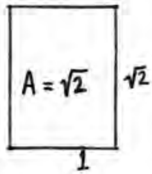
MythBusters, a TV program wanted to fold the paper maximum number of times.

They started with a sheet the size of a football field and they were able to fold it 12 times and used a BULLDOZER to assist them in folding!

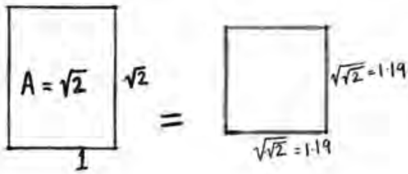
10 Convert A4 Sheet to Square Using All the Paper

Method A

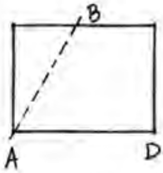
As we saw earlier,the width of an A4 sheet is 210mm and the length is 297mm.
Therefore, area = 210 x 297 = 62,370mm^2.

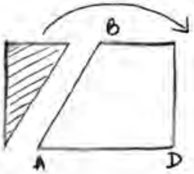


Now, we have to convert the sheet into a square using all the paper. It means that the area of that square would also be 62,370mm^2
Side of the square = $\sqrt{62{,}370}$ = 249.74mm



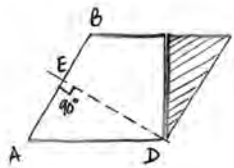
First mark this length from point A to the opposite edge so that the length of line AB is 25 cm. Cut the paper along the line AB.

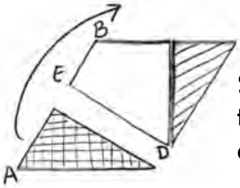




Shift this piece across to the other side.

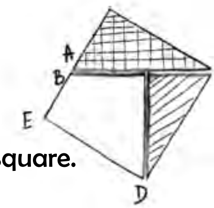
Now mark another line from point D such that the length of DE is also 25cm.
You would see that this line is automatically perpendicular to line AB.



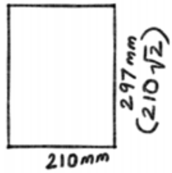


Shift this part also to the other side across the paper.

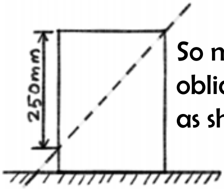
Now all the sides are 25cm and we would get a square.



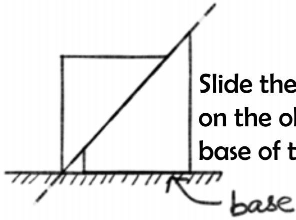
Method B



We know that the area of A4 sheet is 210 x 210$\sqrt{2}$
SO the side of the square with same area will be
$\sqrt{(210 \times 210\sqrt{2})}$ or $210\sqrt{(\sqrt{2})}$ or approx 250mm

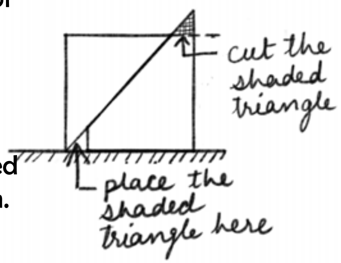


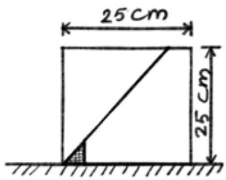
So make an oblique cut as shown.

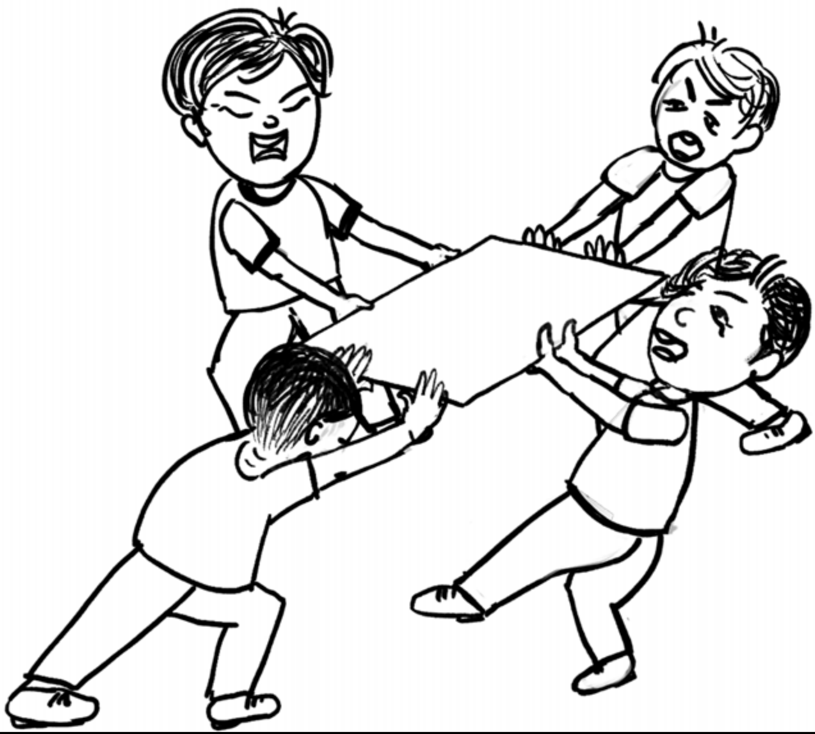


Slide the upper cut portion on the oblique line till the base of the lower cut portion.

Cut the extra triangle of lower part protruding above the upper part and place it in the space above base.
The shape now obtained is a square of side 25cm.



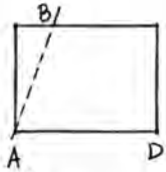


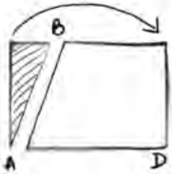


11 A4 Sheet to Rectangle of Given Size

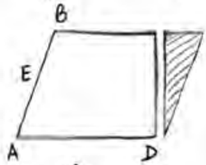
In the previous activity, we made a square from A4 sheet.
This time we will convert the sheet into a rectangle of given dimensions.
Let's make a rectangle of length 23cm.

First mark a point on edge opposite point A so that the length of line AB is 23 cm.
Cut the paper along the line AB.

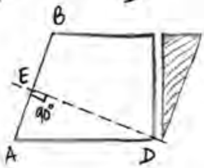




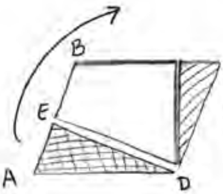
Shift this piece across to the other side.

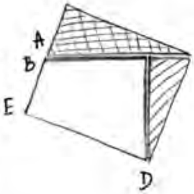


Now mark another line from point D such that it is perpendicular to line AB and cut the paper along that line.



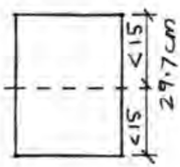
Shift this part also to the other side across the paper.

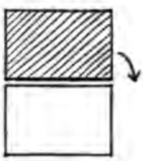


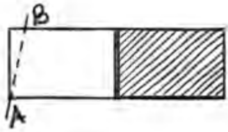


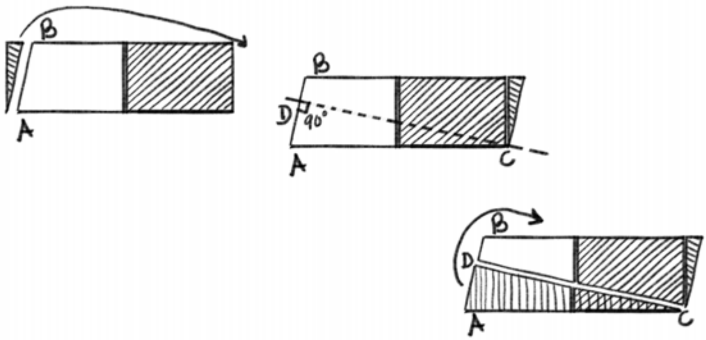
We get a rectangle with one side 23cm or 230mm. We can also predict the other side as we know its area from the previous activity(62,370mm^2.)

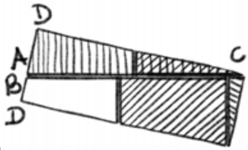
When we make an oblique cut, as we did earlier, we will always get side greater than the width of the paper(21cm). So we can make a rectangle of side 15cm using this method directly because 15 cm is less than 23cm. For this we need to cut the paper in half so that its side becomes less than 15cm. We can cut along the width as shown below.







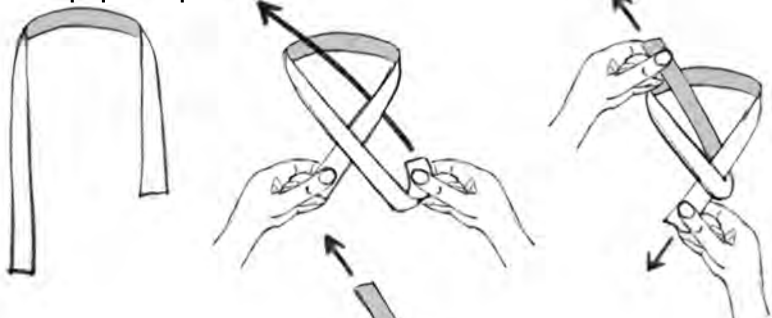




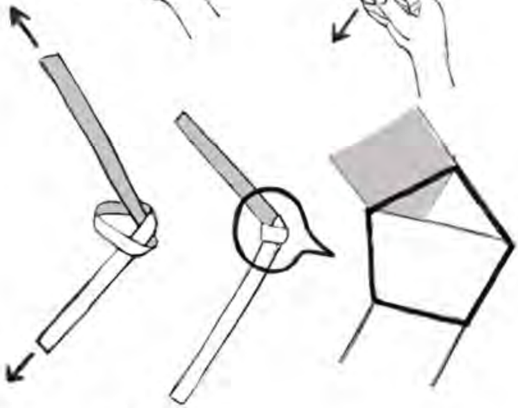
This rectangle would have one side equal to 15cm.
Can you find out the other side without measuring it?

12 Beautiful Paper Folding Knot Technique

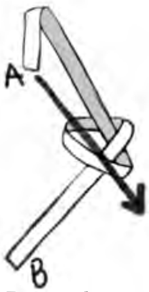
Take a paper strip and make a knot as shown.

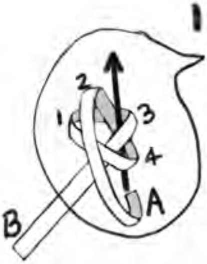


Pull the two ends neatly
to make a pentagon.



To make a 7-sided polygon,
make a knot of pentagon
but do not tighten it.



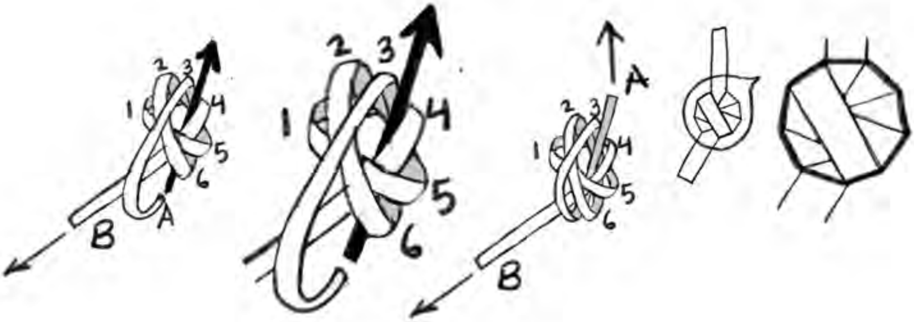


Bring the
top end A
down from
front.

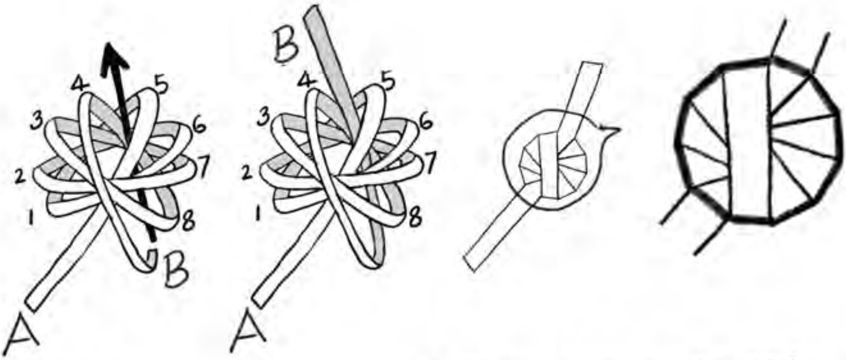
You get four loops.
Insert the end A
through these loops
with 2 on each side.

Pull ends
A and B to
make a knot.

To make a nonagon take the knot of heptagon without tightening it.
Bring the top end A down from front. You get 6 loops.
Insert the end A through these loops with 3 on each side.

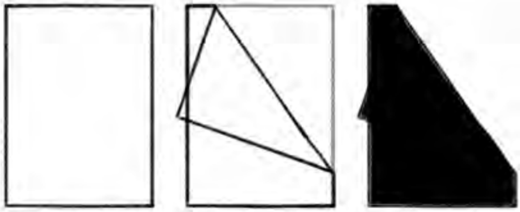


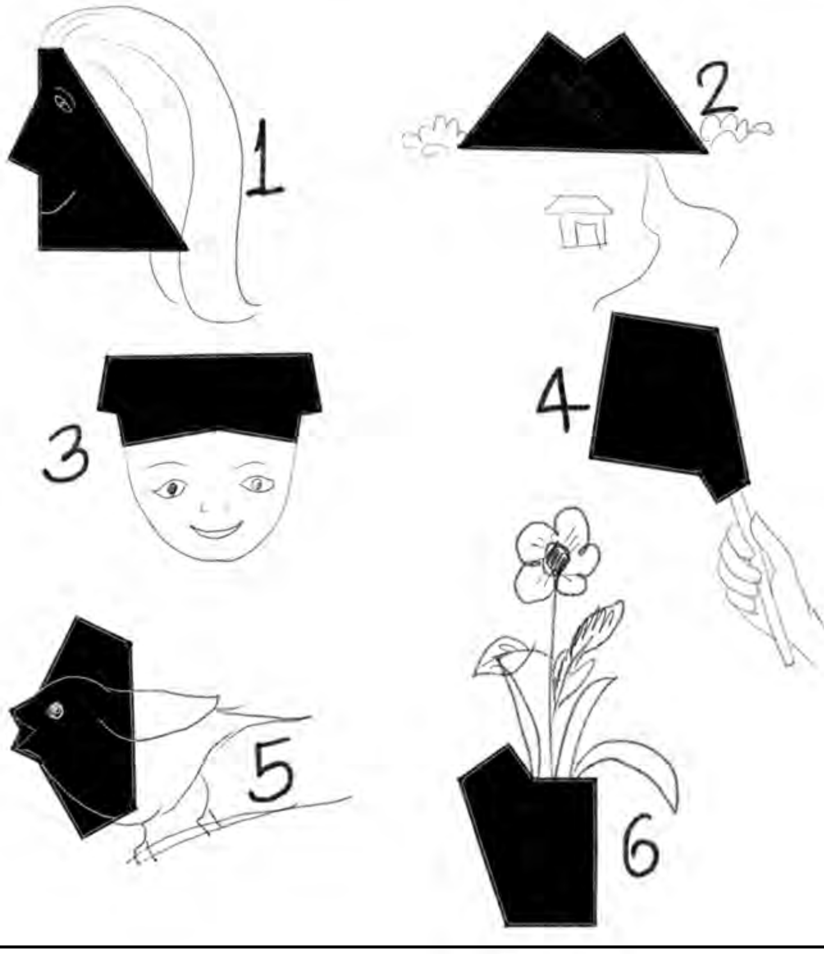
To make 11-sided polygon take the knot of nonagon without tightening it.
Bring the top end A down from front. You get 8 loops.
Insert the end A through these loops with 4 on each side.

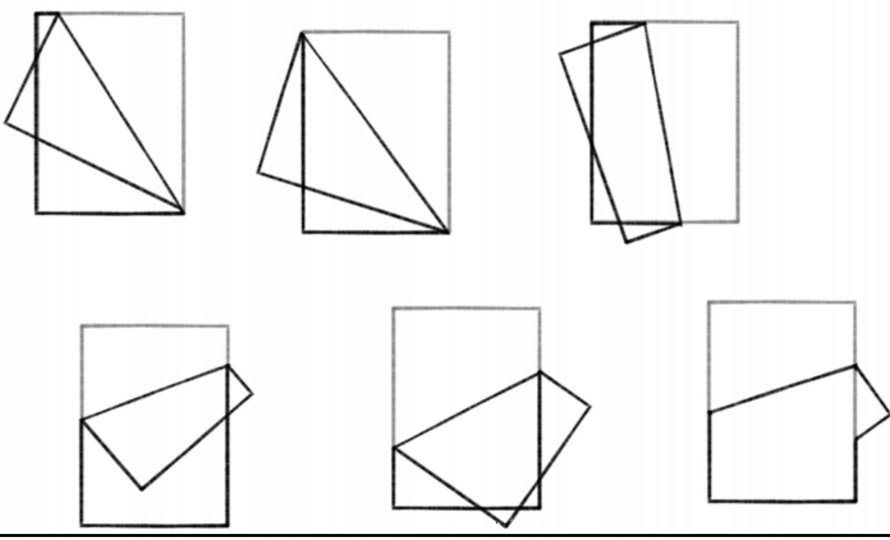


13 Fold A4 Sheet Once to Get Shapes

A sheet of A4 paper is folded *once* and placed on the table flat. Which of these shapes below (region coloured in black) can be made?



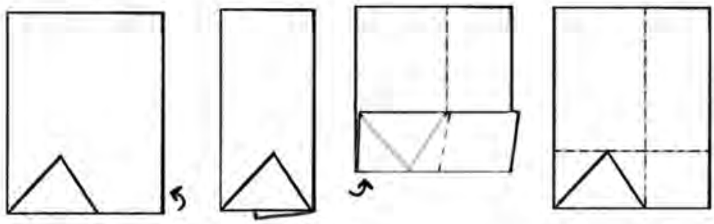




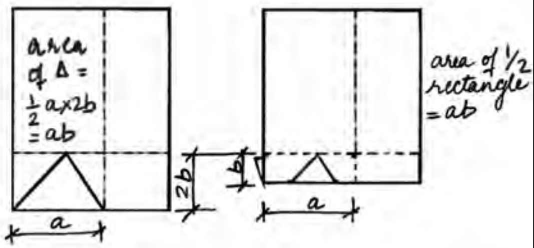
14 Triangle to Square of Same Area

We will convert a triangle into a square of same area.
And that too, with just paper folding and without any measurements.

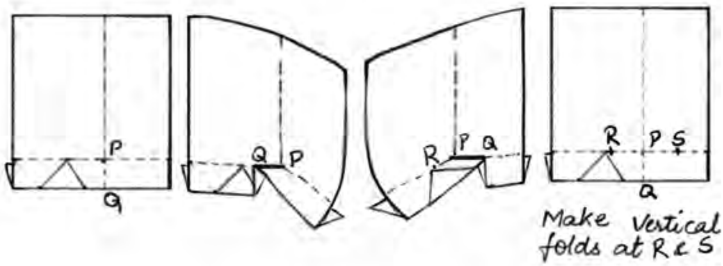
Draw a triangle in the left hand bottom corner of the paper. Now enclose this triangle in a rectangle.



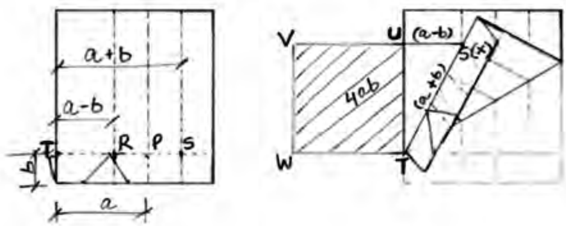
Area of triangle = ½ base x height
= ab
Area of rectangle = base x height
= 2ab
Now if we fold the rectangle into half, we get a rectangle having same area as the triangle.
i,e; a x b



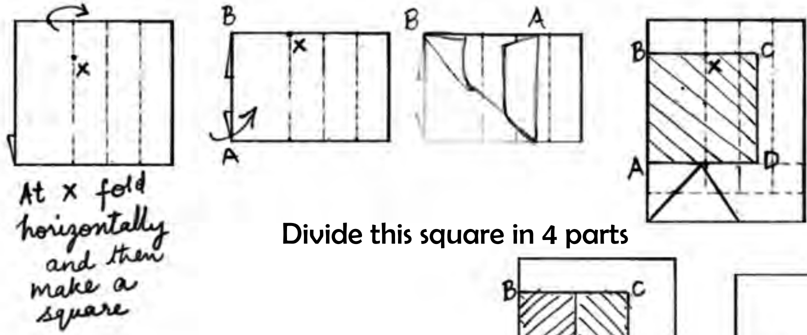
Mark the height of rectangle on both sides of point P (R & S). Draw vertical lines at R & S



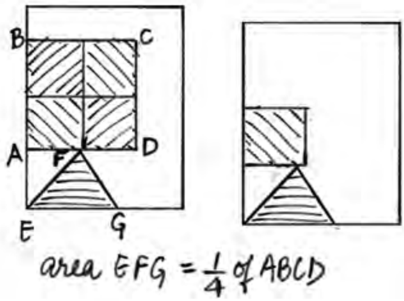
Fold the length TS on vertical line at point R.
Mark that point as X.
Now in triangle TXU,
using pythagoras theorem
$(a+b)^2-(a-b)^2 = 4ab$
or TU = $\sqrt{4ab}$
The square formed using TU as a side will have area 4ab



Now make the square.

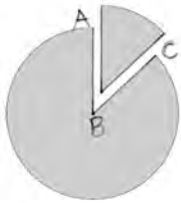


Divide this square in 4 parts



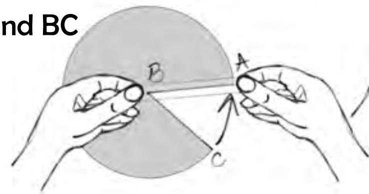
15 Circle to Cone

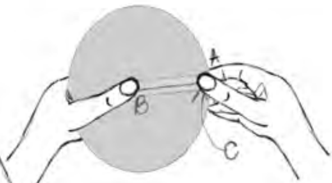
Take a hard paper and cut a circle out of it.



Now cut a sector from this circle.

Join the edges AB and BC with a tape.





Your cone is ready.

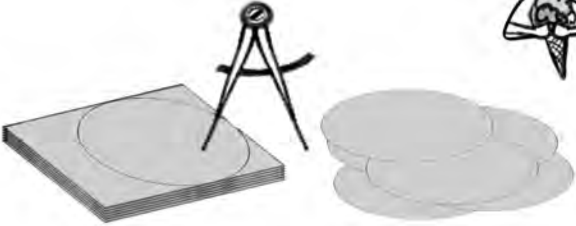
What will happen if you cut a bigger sector?

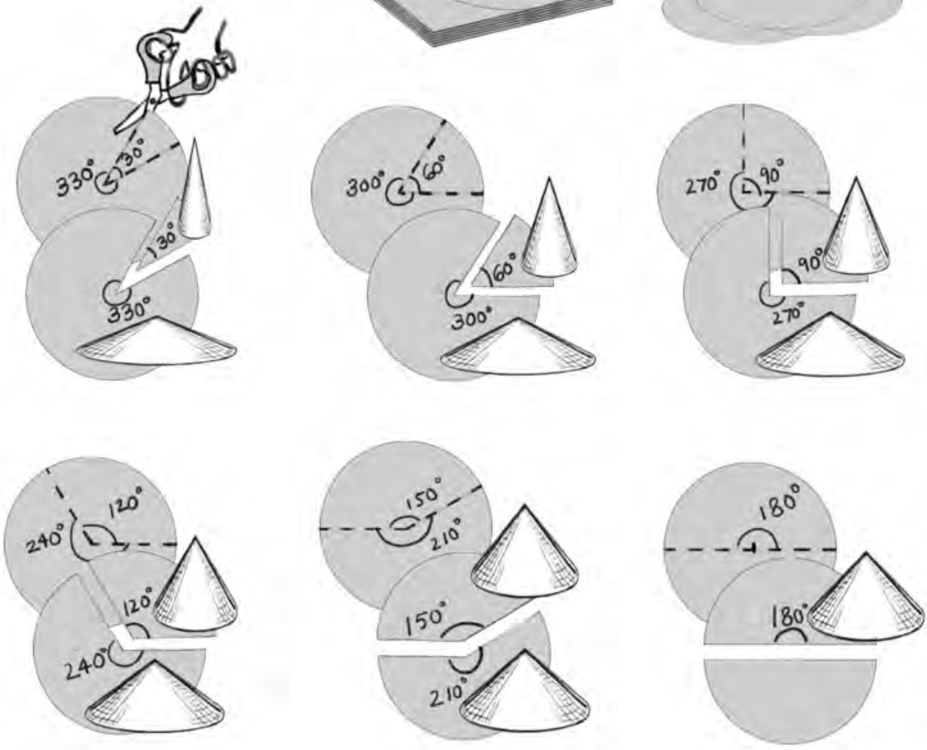
Will the height increase or decrease?
Will the radius of cone increase or decrease?
Cut different size of sectors and observe.


ICE CREAM

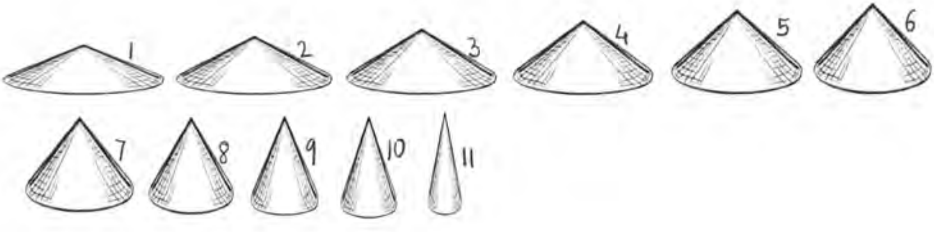
16 Largest Cone from Same Circle

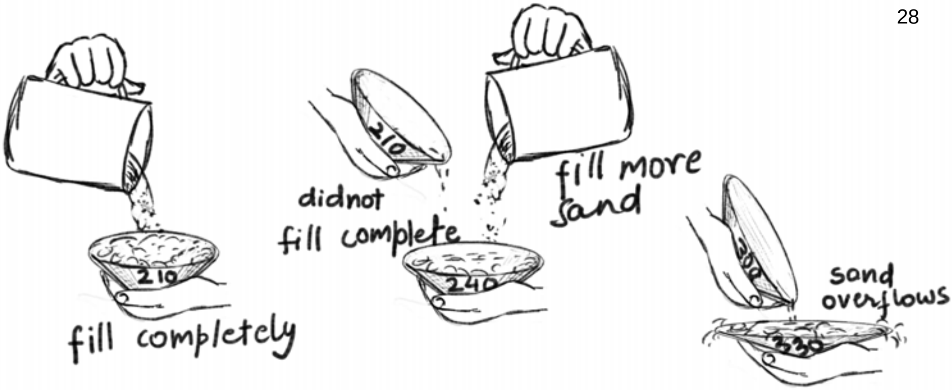
Take hard paper and cut a circle out of it.





Now make cones by cutting out sectors of diffrernt angles, as made earlier.





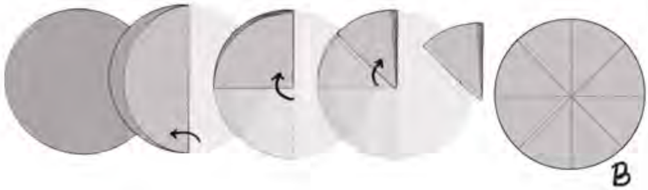
We can observe as we move from cone 1 to 11, the area of the base decreases gradually and the height increases. So which cone has the maximum volume? In other words, which cone would hold the maximum sand?

Take sand in a mug and fill cones one by one. Find out which cone holds the maximum sand.

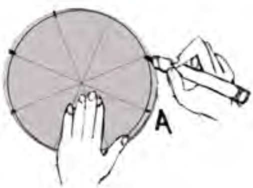
17 Regular Polygons Using a Cone

Take a hard paper and cut a circle out of it(circle A). In the same manner cut a circle of same size with a simple paper(circle B).

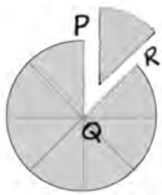
Fold the circle B thrice. Open the paper, you will see that it is divided in 8 equal parts.



Place the circle B on A and mark 8 points on the circumference of circle A.



Now cut sector PQR from this circle.

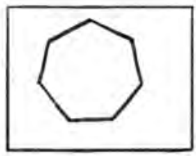


Join the edges PQ and QR with a tape.

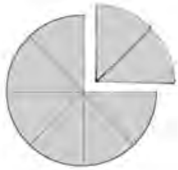
Your cone is ready. It has 7 markings.

Now Place this cone on a paper and mark all 7 points.

Join the 7 points to make a heptagon (7 sided polygon)

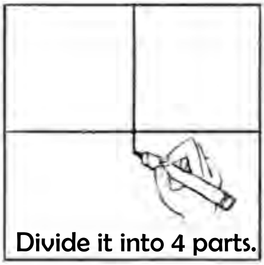


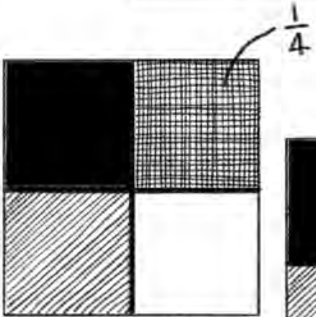
Using the same method, We can make any polygon we want. For example, for making a hexagon, we have to cut two parts from the circle and then make a cone.



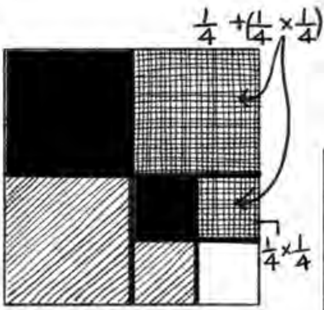
For making polygons with more number of sides, we have to divide the circle in more number of parts. This can be easily done by folding the circle further.

18 Geometric Series

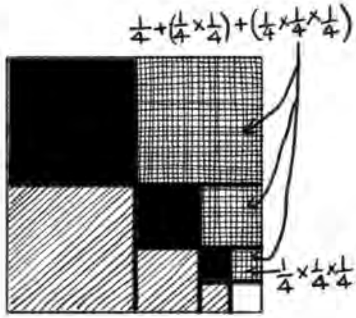




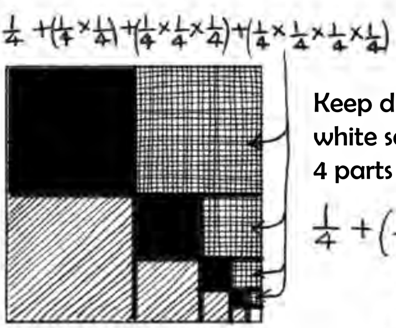
Again divide 1 out of these 4 into 4 parts.



Divide the white square into 4 parts. It means we have divided 1/4 into 4 equal parts again so one part equals 1/4 x 1/4.



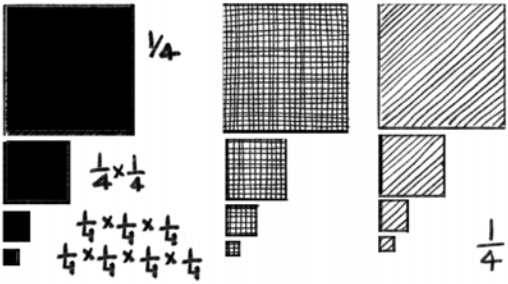
Again divide the white square into 4 parts



Keep dividing the white square into 4 parts till infinity giving a Geometric Series.

$$\frac{1}{4}+\left(\frac{1}{4}\times\frac{1}{4}\right)+\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right)+\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right)\cdots$$

$$=\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}+\frac{1}{4^4}+\frac{1}{4^5}+\frac{1}{4^6}\cdots$$



We can now see that the big square is divided into 3 equal parts denoted by black, checkered and lined parts.

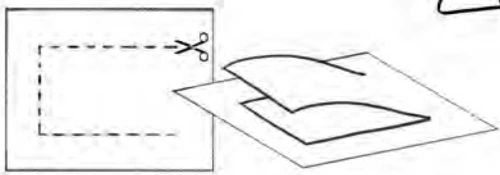
Therefore, if we consider the whole square to be 1, one part becomes 1/3.

$$\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}+\frac{1}{4^4}+\frac{1}{4^5}+\cdots=\frac{1}{3}$$

Carbon dating used by archeologist and paleontologists is a geometrical series!!

19 Knot Magic

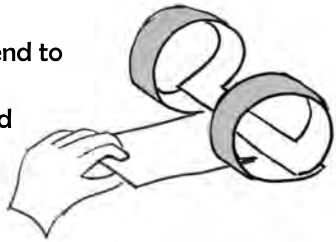
Take an A4 sheet and cut along the dotted line.



Ask your friend to hold the paper as shown.

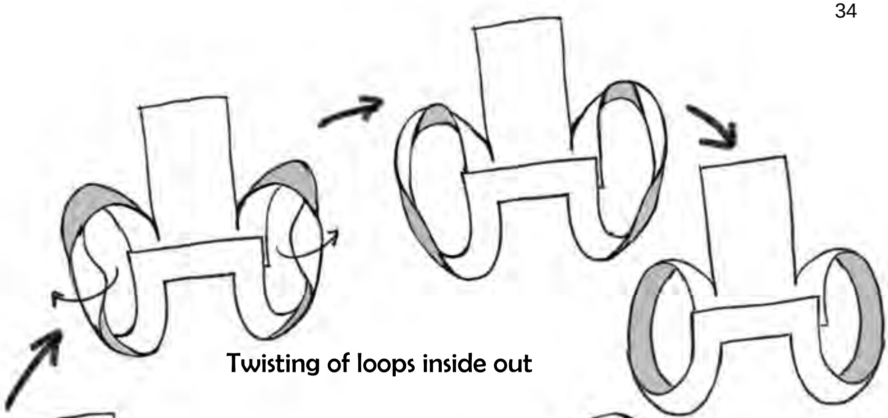
The trick is to bring the paper resting on top of his hand to the bottom without taking the paper out of his hand.

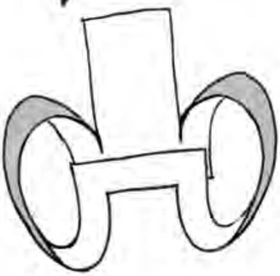
Ask your friend to close eyes.
Now you fold the paper as shown.



Now twist the two hand cuff style rings inside out. Left one anti clockwise and right one clockwise.

Wow! paper handcuffs have gone down!





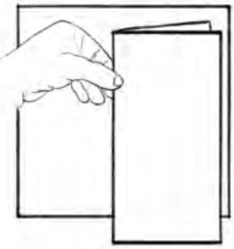
Twisting of loops inside out

Paper handcuffs unfurls.

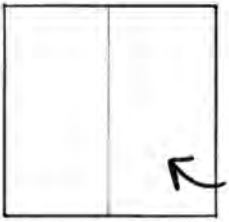
Paper strip is
dangling downwards.

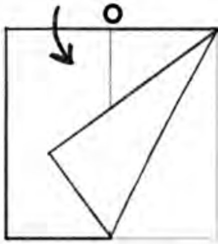
20 Regular Pentagon from Paper Folding

Take a square paper.



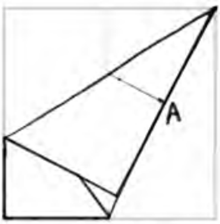
Fold the square paper at the center



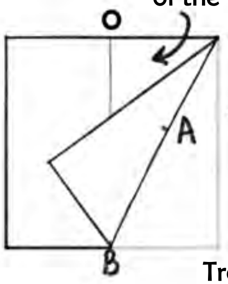


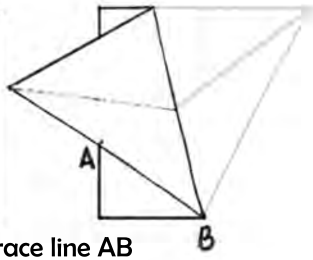
Fold along the diagonal of the rectangle

Mark the half side of the square(O) on this diagonal by placing the side of square on it. Mark this point as A.

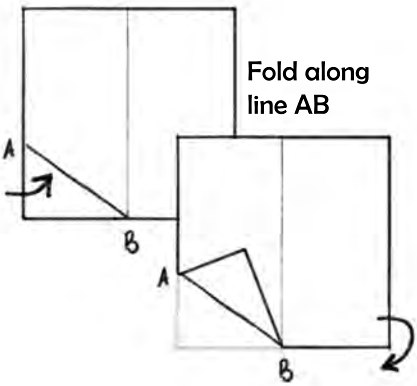


Fold at B such that A lies on the edge of the square



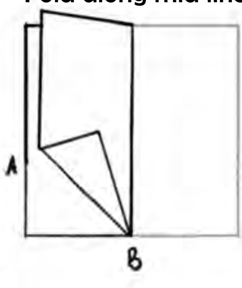


Trace line AB

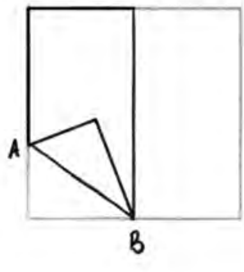


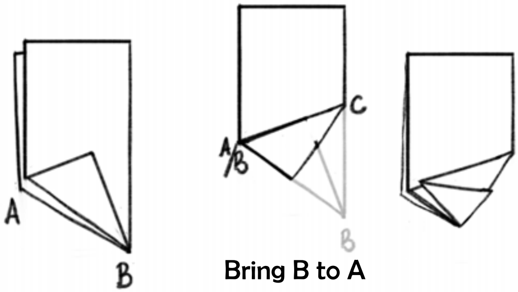
Fold along line AB

Fold along mid line

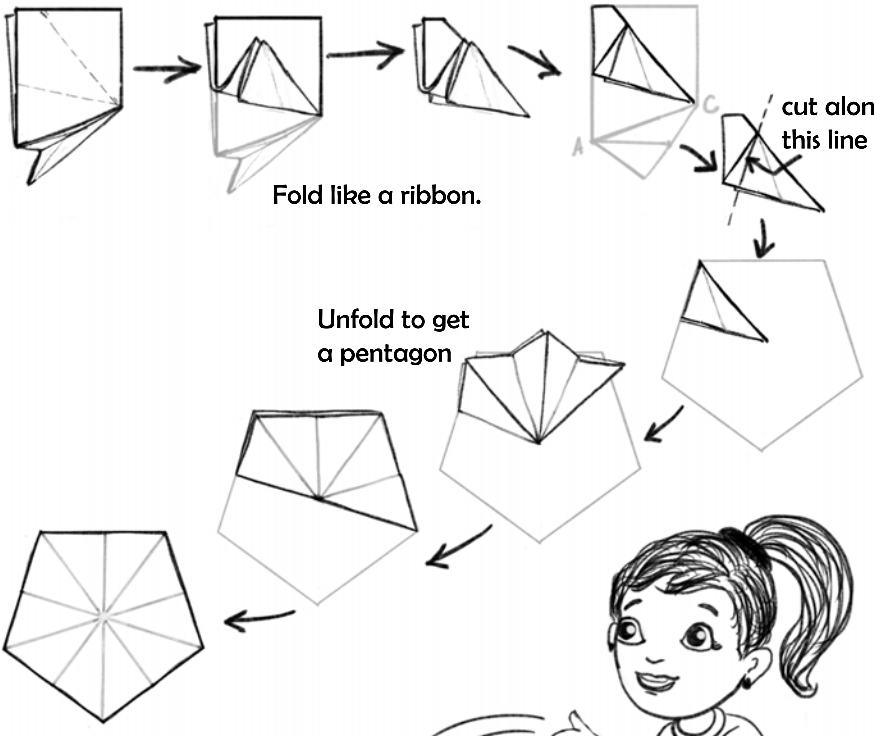


Fold the other side also along AB



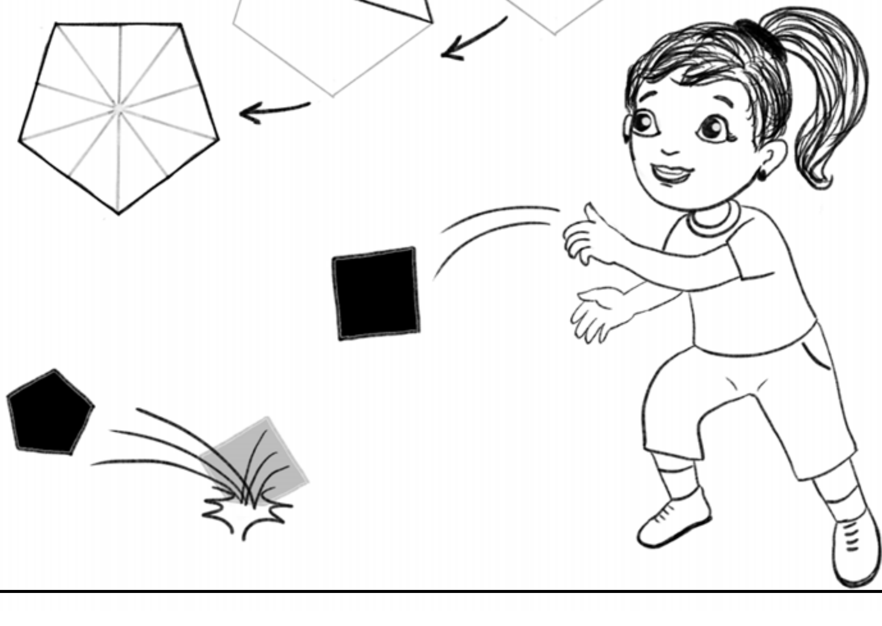


Bring B to A



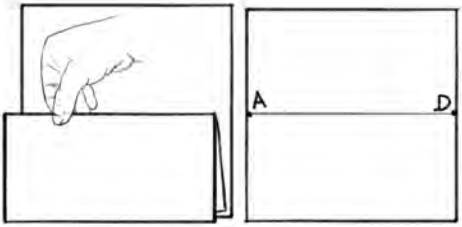
Fold like a ribbon.

Unfold to get a pentagon

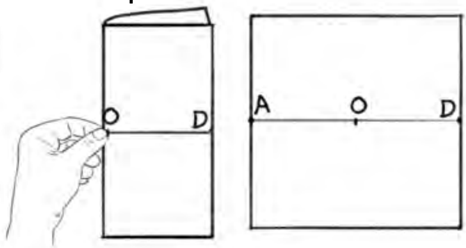


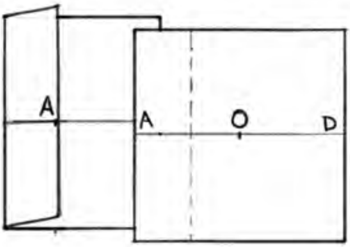
21 Hexagon From Paper folding

Take a square paper and fold it into half and make a line AD on the fold.

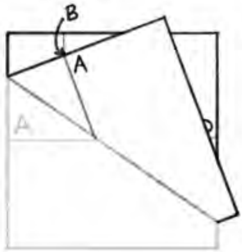


Fold it on the other side to mark the midpoint O of the line.



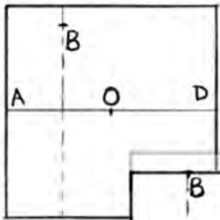


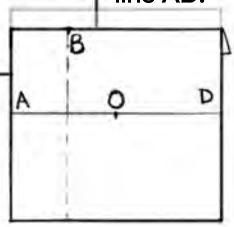
Also mark the quarter line .

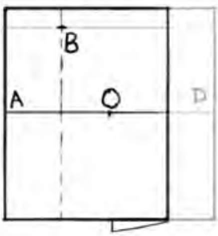


Place point A on the quarter line and mark the point B.

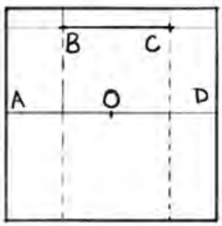
Fold the paper on point B, parallel to the line AD.



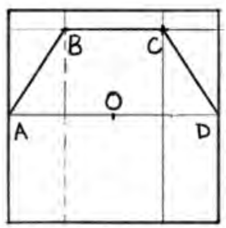




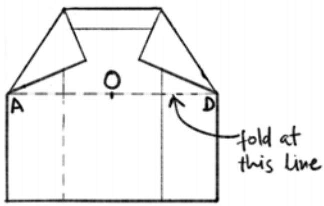
Divide OD in half to get the other quarter line.

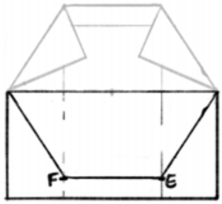


Mark the intersection of quarter line and fold line passing through B as C. Join BC. This is the 1st side of hexagon

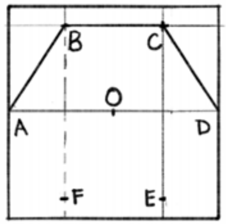


Join A and B, second side join C and D , 3rd side of hexagon.

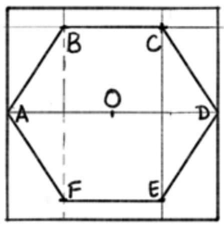


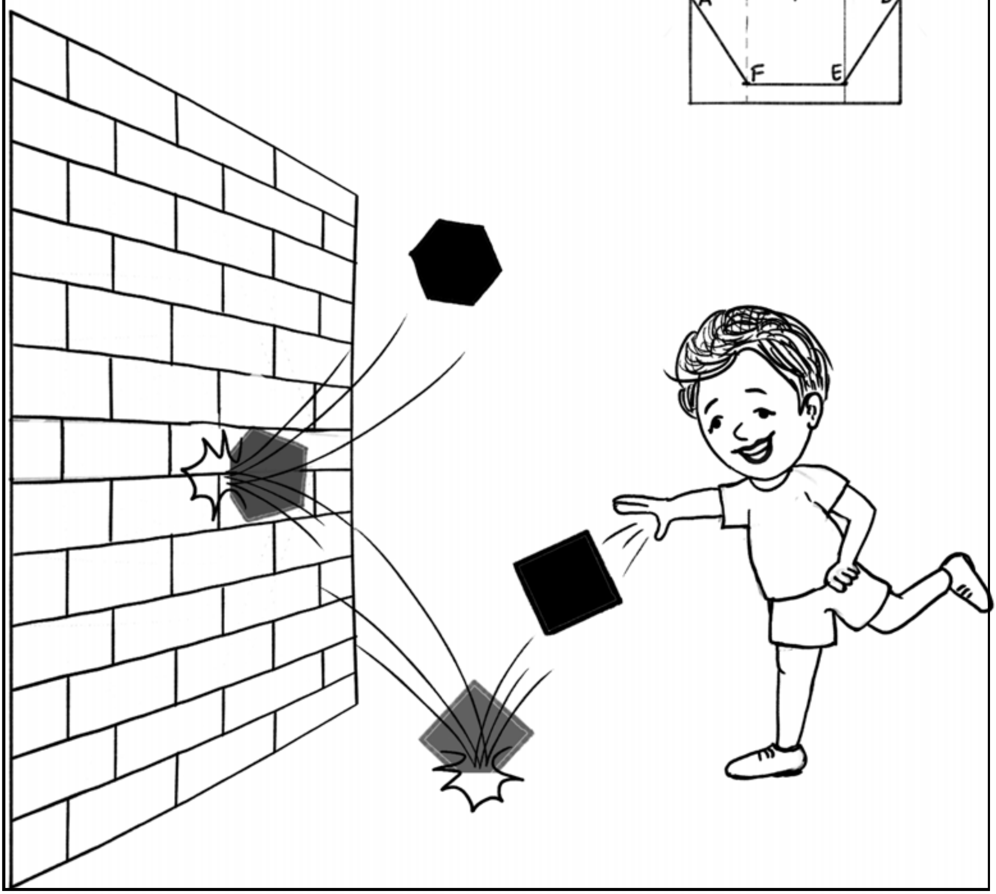


fold paper on lines
AB, BC and CD.

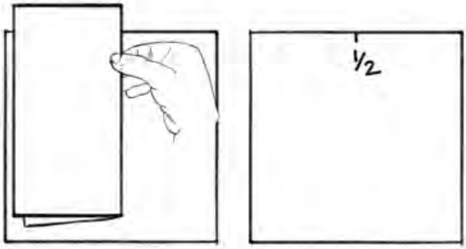


fold along the
line AD and
superimpose
the lines from the
folded portion
to get the
remaining 3 sides.



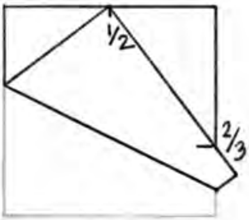


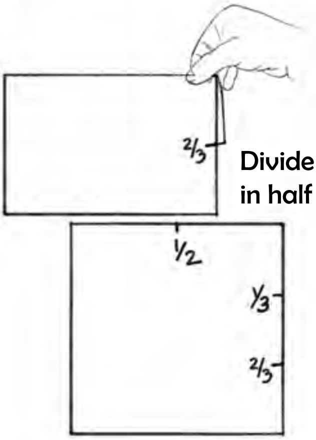
22 Dividing a Line in Equal Number of Parts



Take a square paper and fold it into half. mark the point as 1/2.

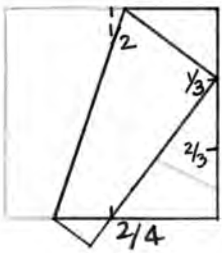
Now bring left corner of the paper on this 1/2 point and fold the paper. The edge cuts the right side of paper at a point. Mark this point as 2/3.

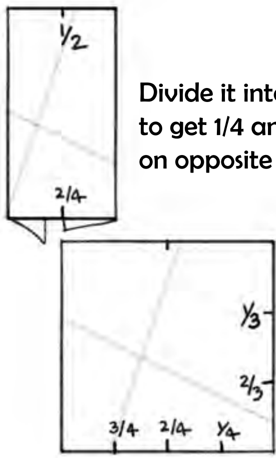




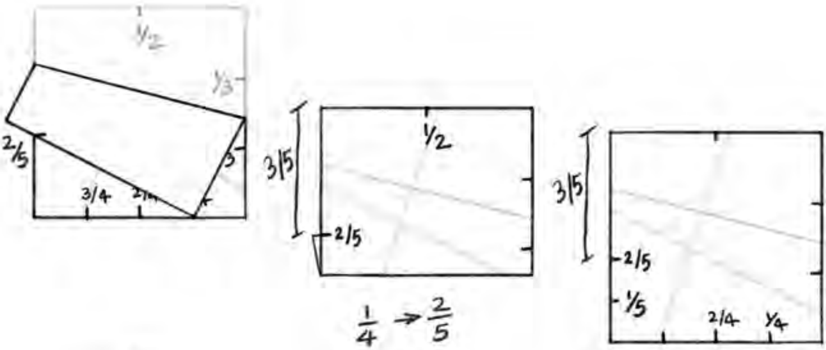
Divide the 2/3 part in half to get 1/3.

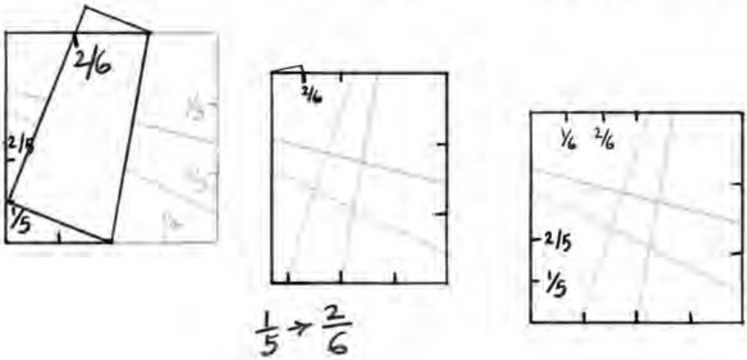
In the same manner bring left corner of the paper on this 1/3 point and fold the paper. The edge cuts the right side of paper at the bottom side. Mark the point as 2/4.





Divide it into half to get 1/4 and 3/4 on opposite sides.

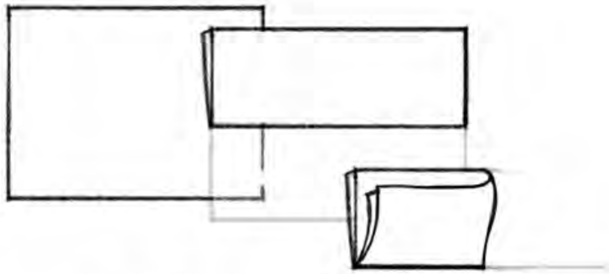




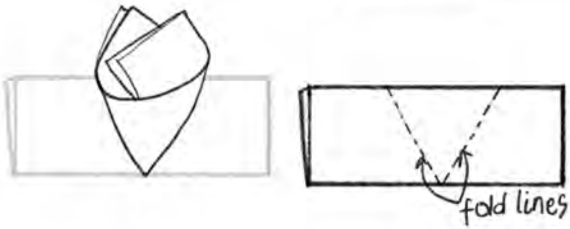
And this way we can go on and on and divide a line in any number of parts.

23 Get Hexagon, Triangle, Pentagon from Single Cuts

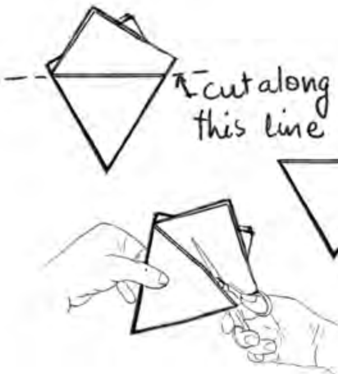
Fold the paper in half. Get the mid point by folding in other direction.

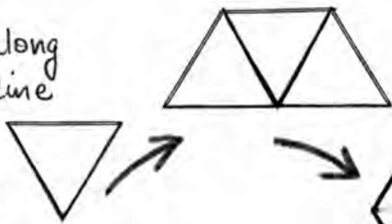


Now fold the paper so as to divide it in 3 equal parts. We get a triangle.



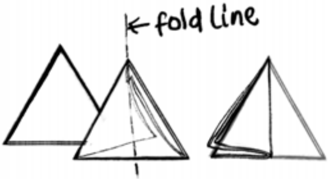
Cut along the line as shown.

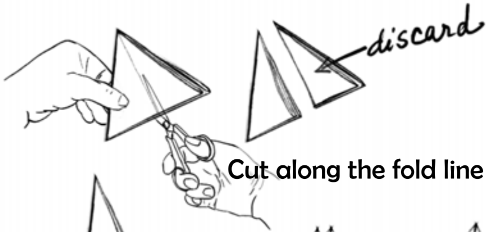




Unfold it to get a hexagon.

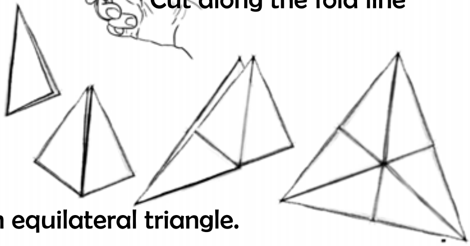
How can we get a pentagon using this method?



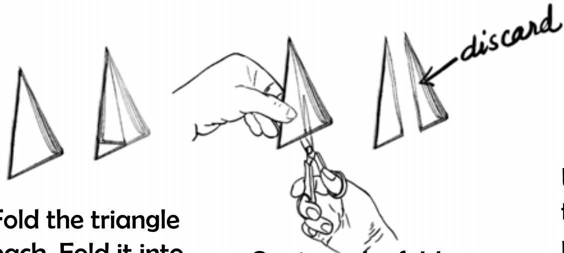


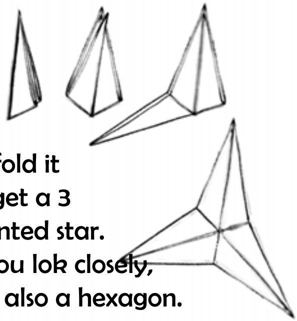
Cut along the fold line

Fold the hexagon back into triangle. Fold it into half as shown.



Unfold to get an equilateral triangle.



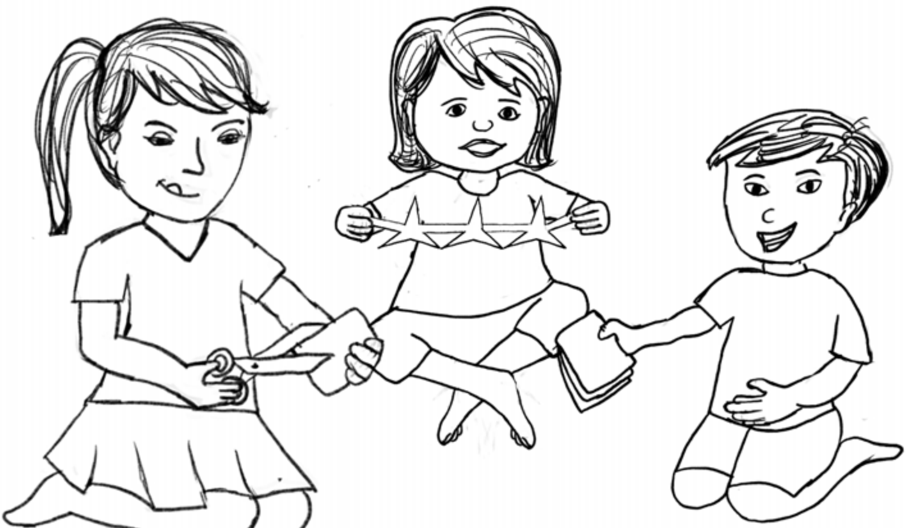


Fold the triangle back. Fold it into half as shown.

Cut it at the fold line so obtained.

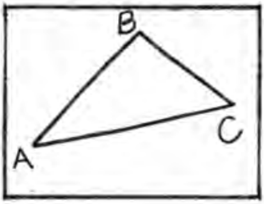
Unfold it to get a 3 pointed star. If you lok closely, it is also a hexagon.

We are geting a triangle if we cut perpendicular to the edge. For all other cases we get a hexagon.

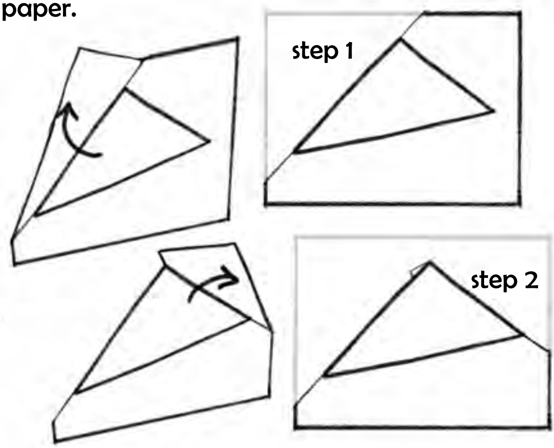


24 Triangle With a Single Cut

We will draw a triangle on an A4 sheet and then cut it out with a single cut.

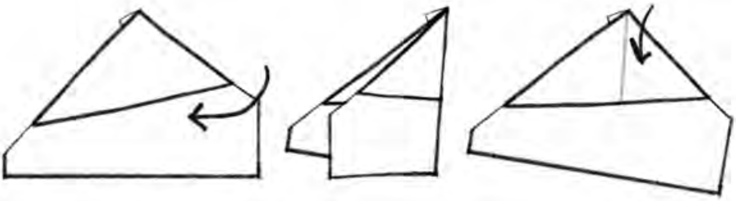


Draw a scalene triangle (all 3 sides unequal) on a paper.

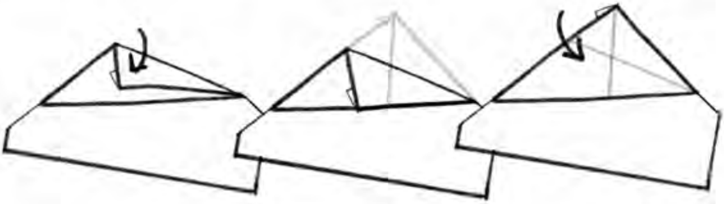


Fold the paper along the sides of the triangle.

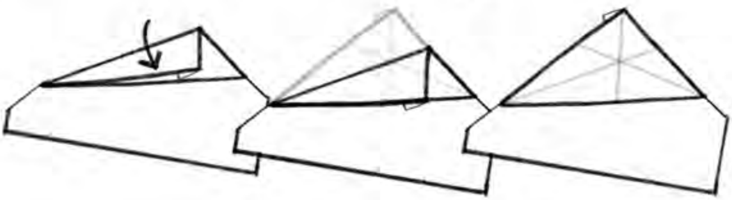
Place one side of triangle on the other (AB on BC) to get an angle bisector.

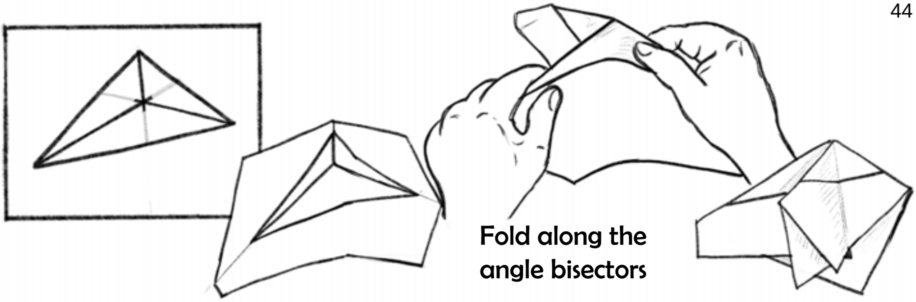


Now put BC on AC and get 2nd angle bisector.

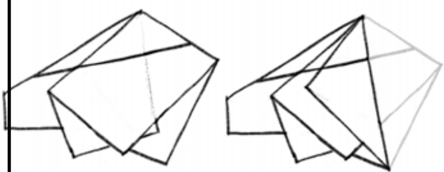


Next put AB on AC to get 3rd angle bisector.

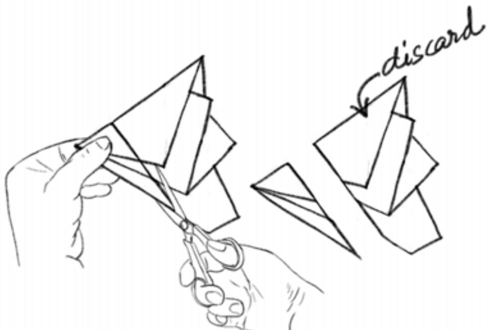




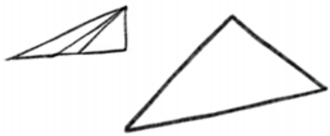
Fold along the angle bisectors



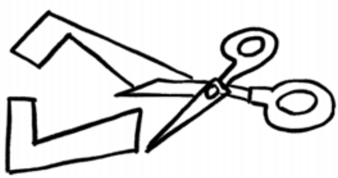
Fold in such a manner that the bases meet.



Cut along the base to cut the whole triangle in one cut.



Unfold to get the drawn triangle



25 Möbius Strip

Let's suppose we have an ordinary band of paper.
What would happen if we cut it in half?
Sure enough, the two parts would fall apart.

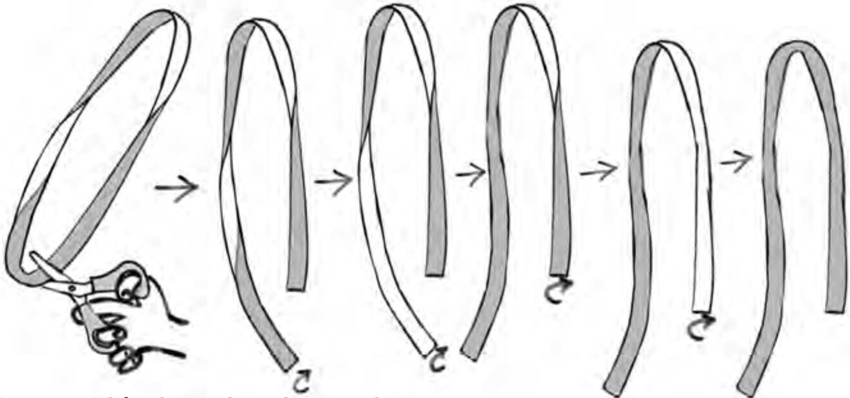
Now, let's make a Möbius strip. We can make a Möbius strip by taking a strip of paper and giving it one turn before joining the two ends.

The special thing about this strip is that it has only one boundary, i.e. if we trace the edge of the strip, we would cover the entire boundary. This is unlike the normal strip where we have two separate disjoint edges.

What would happen if we cut this strip in half? Would we get two parts that would fall apart? Or would we get two entangled pieces?
Let's try.

If we cut the Möbius strip with a single twist in half, we get a single Möbius strip with double length. Now count the number of twists this strip has.

To do this accurately, you can hold the strip in such a way that the all the twists are in the upper half and the lower half has no twists. Now cut the lower half and slowly untwist the upper half.



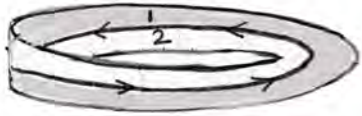
You would find out that the strip has 4 twists.

Neck ribbon configured as Möbius strip allows it to fit comfortably around the neck while the medal lies flat on the chest.

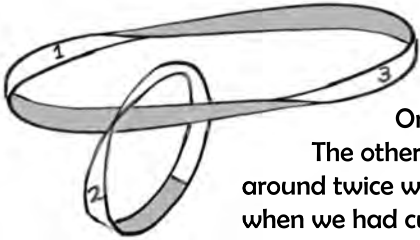
26 Single Möbius Strip (Half Twist) Cut in 1/3

In the last activity, we had cut the Möbius strip in half.
What happens when we cut the strip in 1/3rd. Let's try.

When we cut the strip in half, we end up at the same point when we complete one turn.



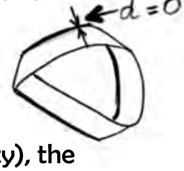
But when the cut is 1/3rd (or any other value than half), we end up on the different side after a complete trip of the strip. And in order to separate the part we are cutting from the original strip, we have to take one more trip.



When we complete these two trips, we get two strips which are entangled. One is our original strip with a single twist. The other one is twice in length because we went around twice while cutting it and with 4 twists, just like when we had cut it in half!!

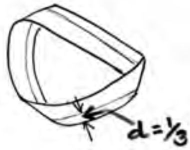
Let's say that the distance of the cut is d from both sides. The value of d can vary from 0 to ½.

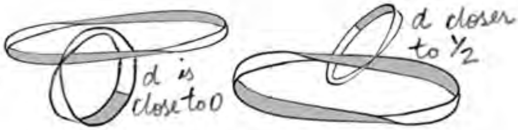
When d is zero, we would just brush past the paper without cutting anything, and obviously, we would have the original strip in the end.



When d is ½ (which we had seen in the last activity), the middle part becomes zero and we get a single strip twice the length of the original strip.

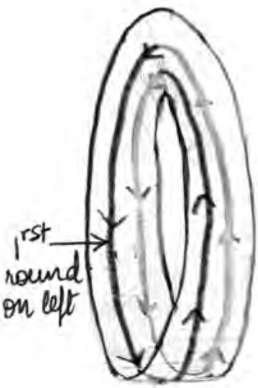
You can try with a value of d closer to zero and one closer to ½ to see the effect on the resulting strips. When d is close to 0, the strip having 4 turns is narrow and the strip with 1 twist is wide. And when d is closer to ½, the strip having 1 turn is narrow and the strip with 4 turns is wide.



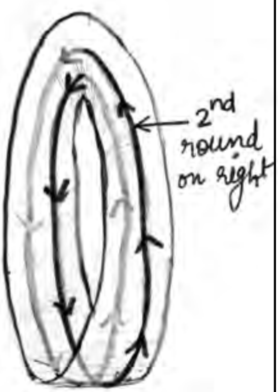


Now, why are they entangled?

Let's trace the side part and see where it is in relation to the middle part.



If initially it runs left of the middle part, after one round it comes to the right side of the middle strip.
So we can say that it has crossed once over the strip.
Now again, by the time we cover our second trip, it comes back to the left side of the middle strip.

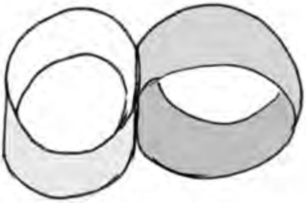


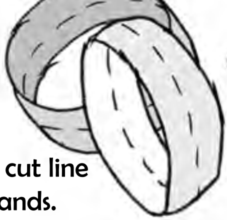
Thus the narrower strip has crossed the middle strip twice and entangled itself around the middle one.

27 Double Möbius with Axis Perpendicular

We have learned to make Möbius bands in the previous activity. Now we will join two bands together and then see what happens when we cut them.

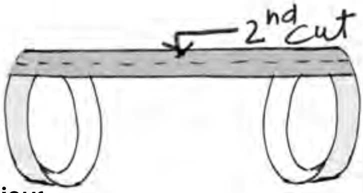
Join two bands with no twist to each other such that their axes are perpendicular.

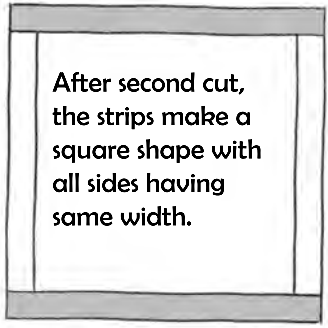




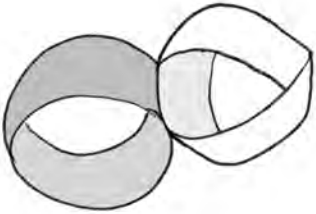
Mark the half cut line on both the bands.

First cut one band. The band will split into 2, each having width half the previous. They will be joined with a strip(cut 2nd band)



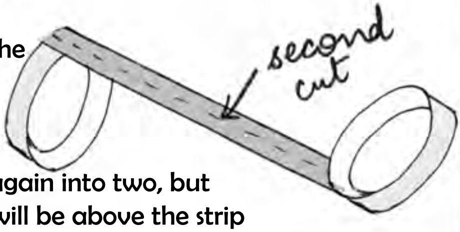


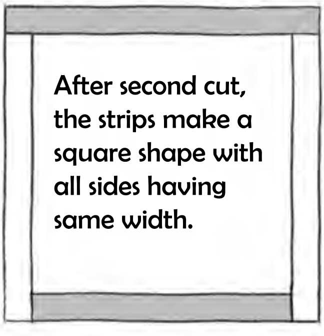
After second cut, the strips make a square shape with all sides having same width.



Now join a normal band (no twists) and a Möbius strip with their axes perpendicular. Mark the half cut line on both the bands

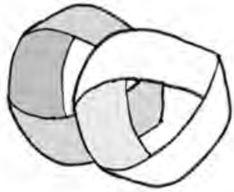
First cut the normal band. The band will split again into two, but one ring will be above the strip and the other one below.





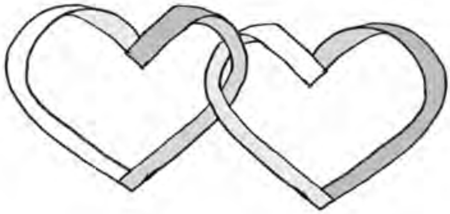
After second cut, the strips make a square shape with all sides having same width.

Now join two Möbius bands with no twist to each other such that their axes are perpendicular.

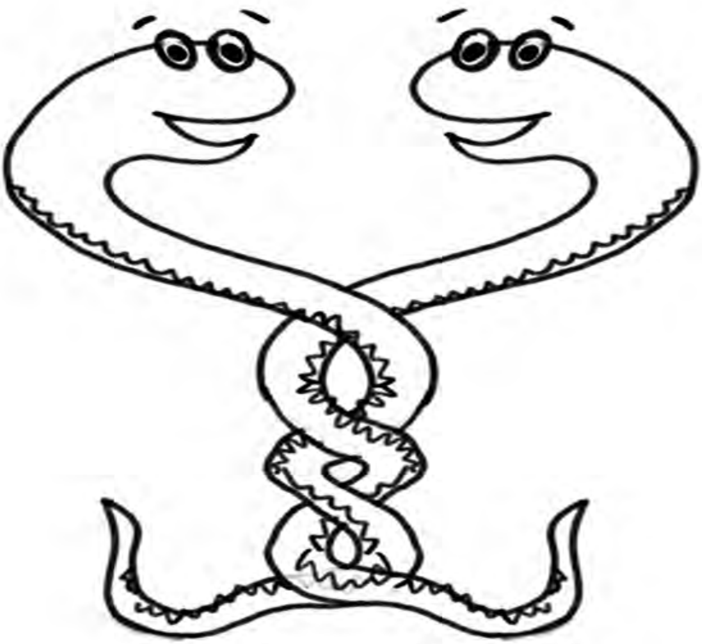


Mark the half cut line on both the bands.

The bands will split into two parts, each having width half the previous. Both are identical hearts.

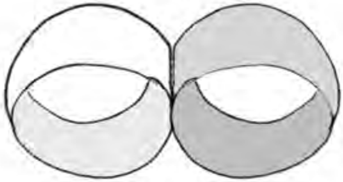


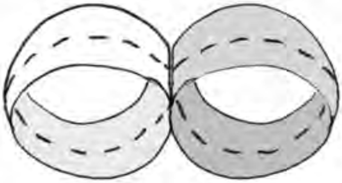
In this case, what direction did you twist the strips? If we twist both the Mobius strips in opposite direction, we get two separate hearts. But if they are twisted in the same direction, we get two entangled hearts!



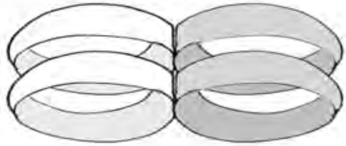
28 Double Möbius with Parallel Axis

First join two bands with no twist to each other such that their axes are parallel.





Mark the half cut line on both the bands and cut them along the line.

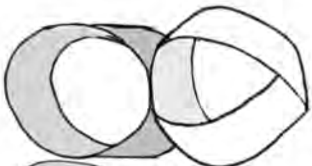


The bands will split into two, each having width half the previous.

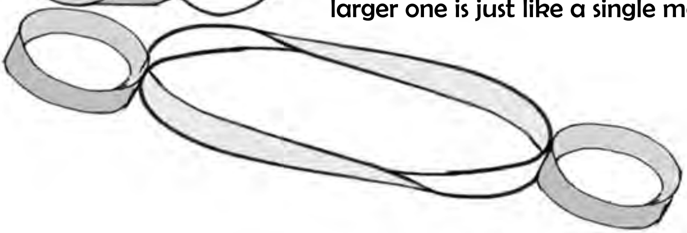
What will happen if we cut the bands at 1/3rd width?

In this case, nothing intersting happened. Let's try other combinations.

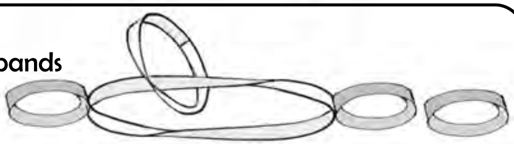
Now join a normal band (no twists) and a Möbius band, such that their axes are parallel. Mark the half cut line on both the bands and cut them along the line.

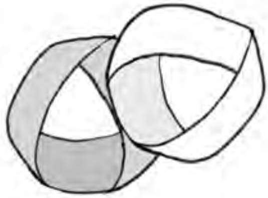


The bands will split into three parts, each having width half of the original bands.
The smaller ones will be without twist and the larger one is just like a single mobius split into half.

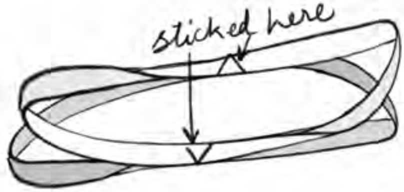


Try cutting the the bands at 1/3rd width?



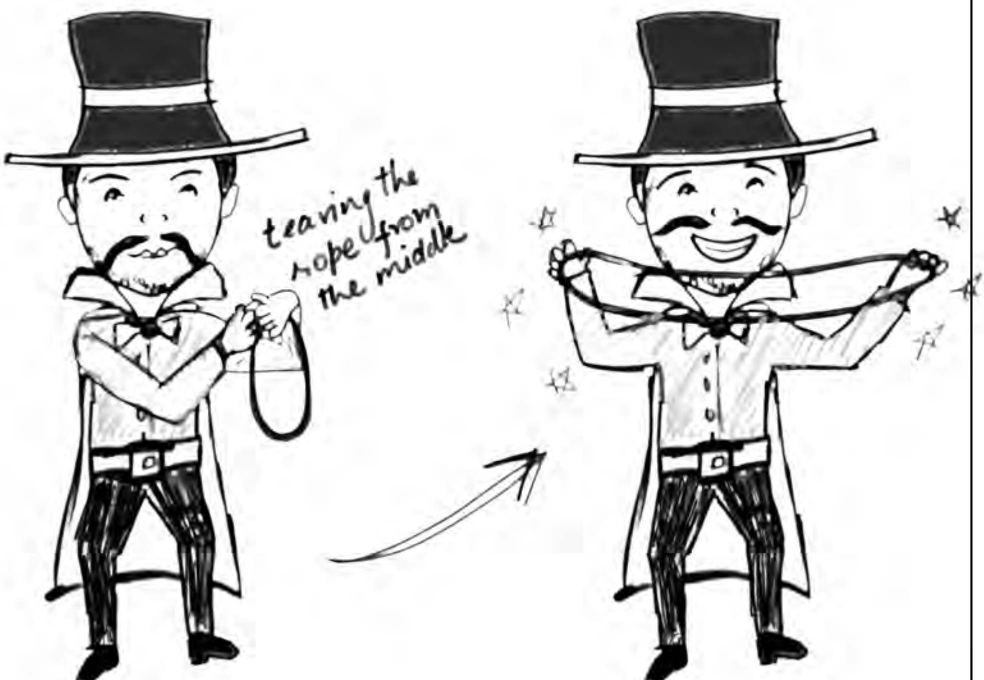


Let's join now two Möbius bands, again keeping their axes parallel. Mark the half cut line on both the bands and cut them along the line.



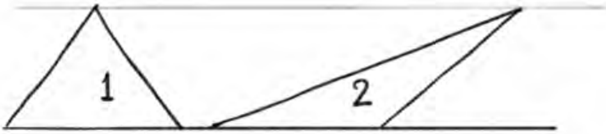
The bands will split into two parts, each having width half the original bands. Both are identical and placed one above the other. They are just like a single mobius split into half.

Try cutting the the bands at 1/3rd width? They will be just like two single mobius cut at 1/3rd but sticked to each othjer.

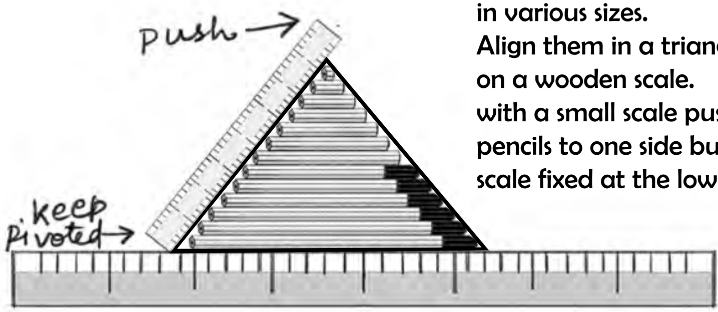


29 Area of a Triangle

The different triangles having same base and height will have same area.

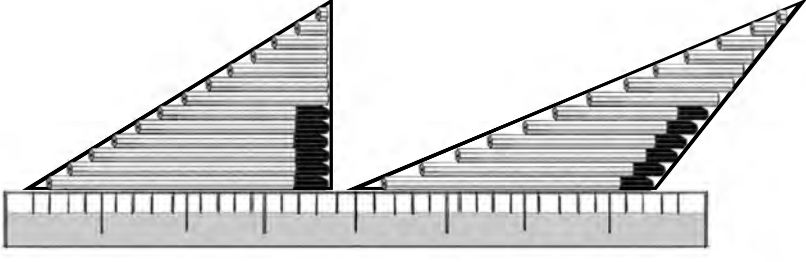


Triangle 1 and 2 have same area although they look so different.

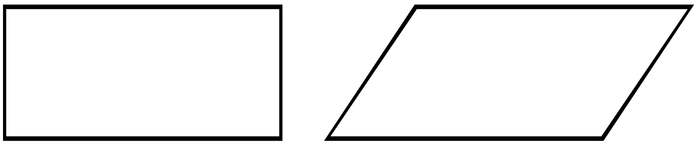


Take a few pencils and cut them in various sizes.
Align them in a triangular manner on a wooden scale.
with a small scale push all these pencils to one side but keeping the scale fixed at the lower part.

With the same pencils and its stubs you can get these triangles.

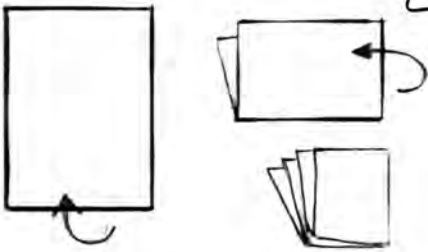


Do you think a rectangle and a parallelogram with same base and height will have same areas? Find out in the same manner.

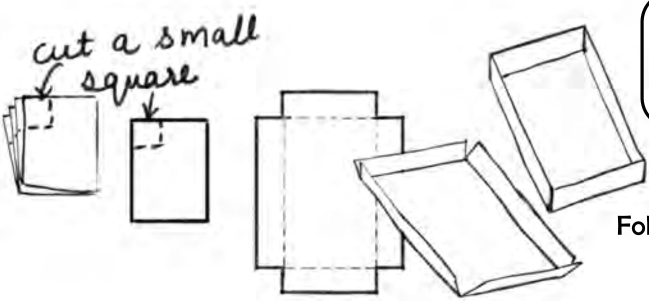


30 Largest Box from A4

Fold a sheet of A4 paper twice to get 1/4th its size.



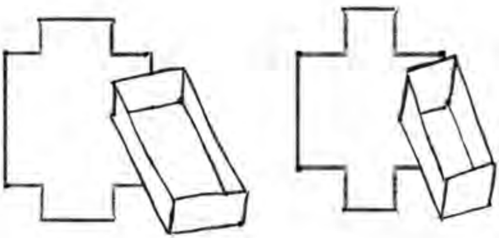
Cut a small square from this folded sheet such that squares are cut from all the four corners.



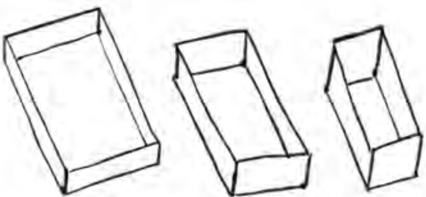
Can you cut all the four squares with a single cut?

Fold it to make a box

In the same manner make other boxes, increasing the size of square successively. Stick the sides with the tape.



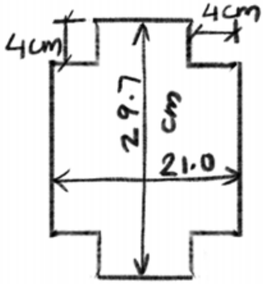
We would see that gradually the area of the base decreases and the height of the box increases as we increase of the cut square.

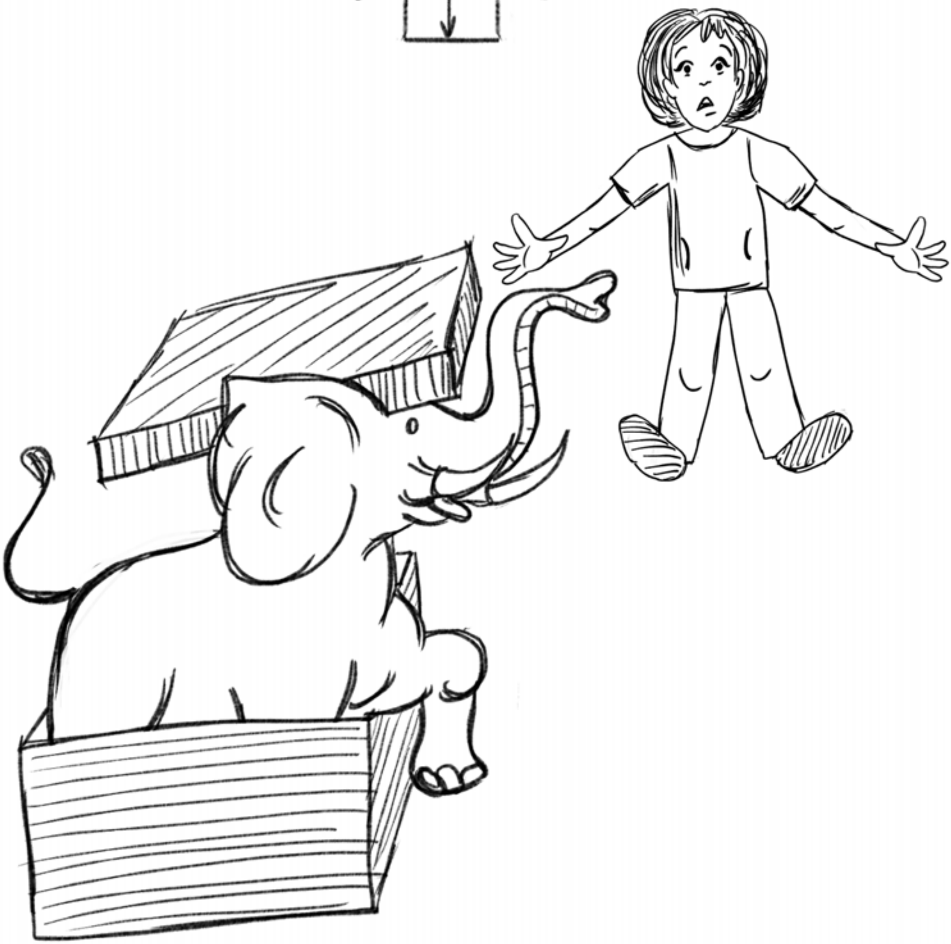


Which of these boxes has the maximum volume?

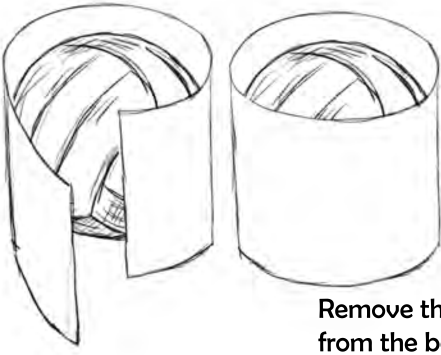
Fill the boxes with sand and find out which box can take maximum amount of sand. And we can say that this box has the maximum volume.

The size of such box comes out to be maximum when a square of approximately 4 cm length is cut in case of an A4 sheet.





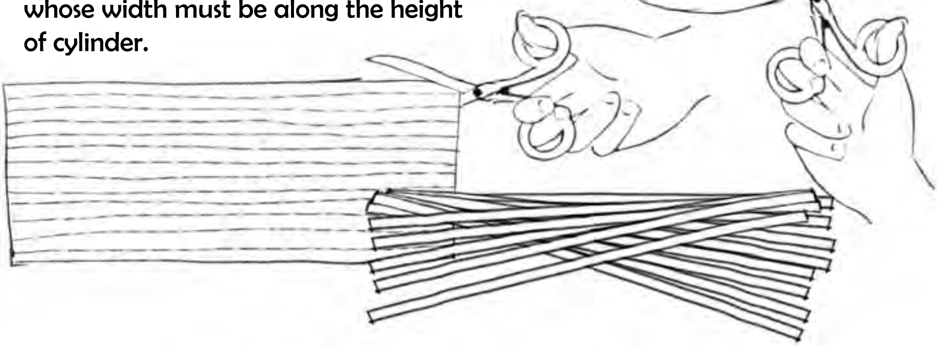
31 Cylinder and Sphere Have Same Areas

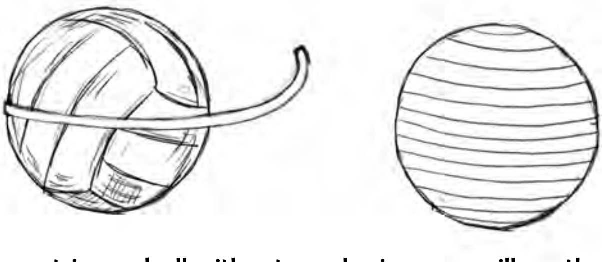


Take a paper and and make a cylinder around a ball. The height of cylinder must be equal to the height of ball and cylinder should touch sphere at its equator.

Remove the cylinder from the ball and cut it longitudinally.

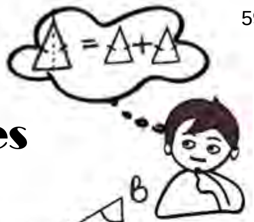
We get a rectangular shape of paper. Now cut this paper in very thin strips whose width must be along the height of cylinder.





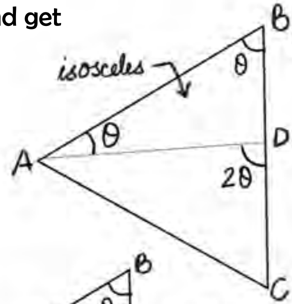
Stick all these strips on ball without overlaping. you will see that the strips cover the sphere completely. This shows that the curved surface of cylinder is equal to the surface area of sphere.

There will be some surface of sphere which wont be covered. This happens because the paper gets crumpled when we stick it on sphere which gets wasted otherwise it could have been used for the left out space. The problem can be solved if we cut the strips of width nearing to zero.



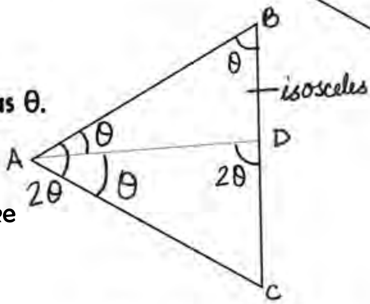
32 Cut Isosceles Triangle Get 2 Isosceles Triangles

Can we cut an Isosceles triangle in 2 parts and get 2 isosceles triangles? Well we can do this in some special cases. Lets find them.



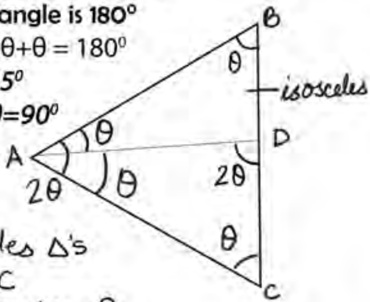
In the triangle ABC, let's assume angle B and angle BAD as θ. So the exterior angle ADC of triangle ABD becomes 2θ. Triangle ABD is isosceles.

A) Let us assume angle DAC as θ.

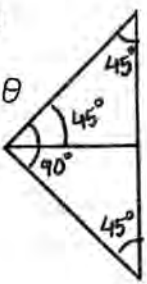


To make triangles ABC and ACD isosceles we need to make either angle C as θ or 2θ

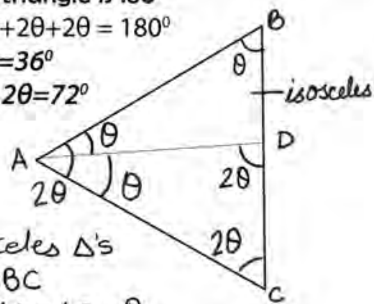
Case 1: angle C = θ
Sum of interior angles of a triangle is 180°
So $\theta+2\theta+\theta = 180^0$
or $\theta=45^0$
and $2\theta=90^0$



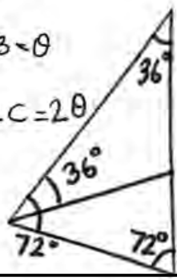
Isoceles Δ's
Δ ABC
$\angle B = \angle C = \theta$
ΔABD
$\angle BAD = \angle B = \theta$
Δ ACD
$\angle DAC = \angle C = \theta$



Case 2: angle C = 2θ
Sum of interior angles of a triangle is 180°
So $\theta+2\theta+2\theta = 180^0$
or $\theta=36^0$
and $2\theta=72^0$

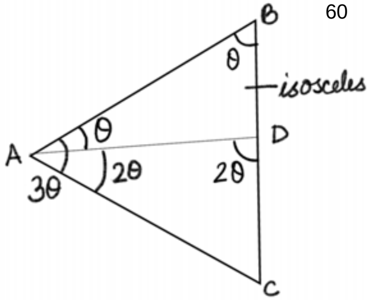


Isoceles Δ's
Δ ABC
$\angle A = \angle C = \theta$
ΔABD
$\angle BAD = \angle B = \theta$
Δ ACD
$\angle ADC = \angle C = 2\theta$



B) Let us now assume angle DAC as 2θ.

To make triangles ABC and ACD isosceles we need to make either angle C as θ or 3θ



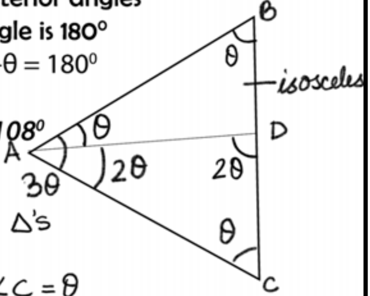
Case 3: angle C = θ

Sum of interior angles of a triangle is 180°

So $\theta + 3\theta + \theta = 180^0$

or $\boldsymbol{\theta = 36^0}$

and $\boldsymbol{3\theta = 108^0}$



Isoceles Δ's

Δ ABC

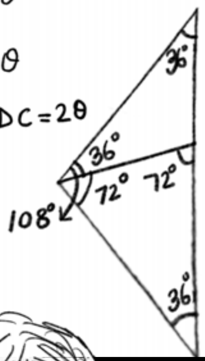
∠B = ∠C = θ

ΔABD

∠BAD = ∠B = θ

Δ ACD

∠DAC = ∠ADC = 2θ

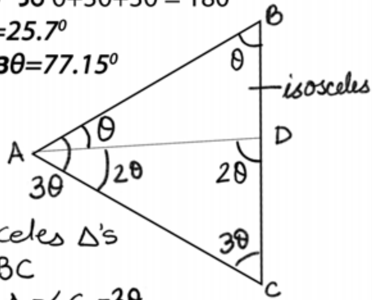


Case 4: angle C = 3θ

Sum of interior angles of a triangle is 180° So $\theta + 3\theta + 3\theta = 180^0$

or $\boldsymbol{\theta = 25.7^0}$

and $\boldsymbol{3\theta = 77.15^0}$



Isoceles Δ's

Δ ABC

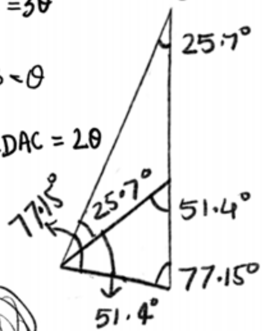
∠A = ∠C = 3θ

ΔABD

∠BAD = ∠B = θ

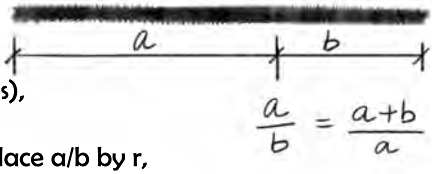
Δ ACD

∠ADC = ∠DAC = 2θ



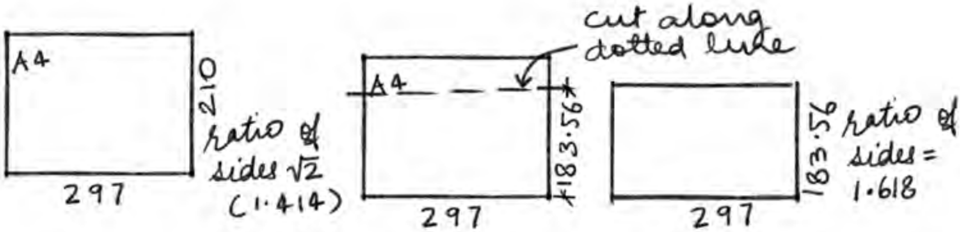
33 Golden Ratio in Rectangle

Two quantities are said to be in golden ratio in their ratio is the same as ratio of their sum to the larger of the two quantities.

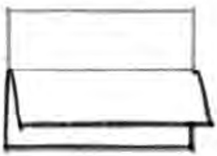


So if a and b are in golden ratio
(and a is the larger of the two quantities),
it means that a/b = (a+b)/a
Let's try and solve this. If we denote replace a/b by r,
the above equation becomes:
$r=1 + 1/r$ or $r^2 = r+1$
Solving this, we get $r= \frac{(1+\sqrt{5})}{2} = 1.618$
This ratio is denoted by the the greek alphabet φ.

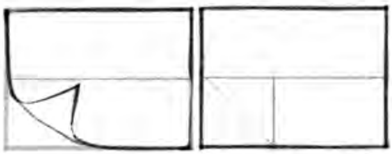
As we had seen in the first activity, the ratio of sides of an A4 sheet is √2 which is equal to 1.414. If we want to convert it into a golden rectangle, the ratio of its sides should be 1.618. If we keep the length unchanged at 297 mm, we have to reduce the breadth to increase the ratio of sides from √2 (1.414) to φ (1.618). The new breadth should be 297/1.618 = 183.56



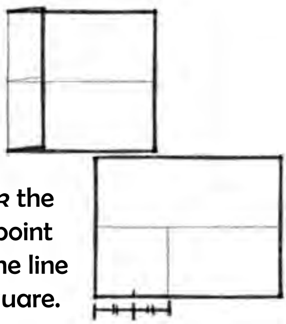
Lets make a golden rectangle.



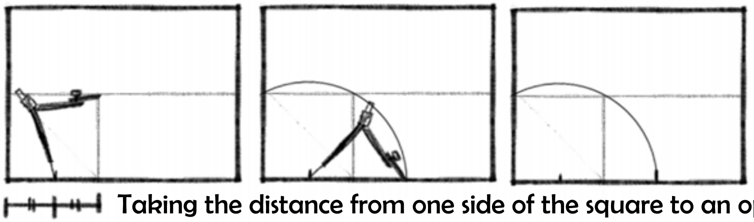
Fold the A4 paper into half.



Mark a square by folding paper diagonally.

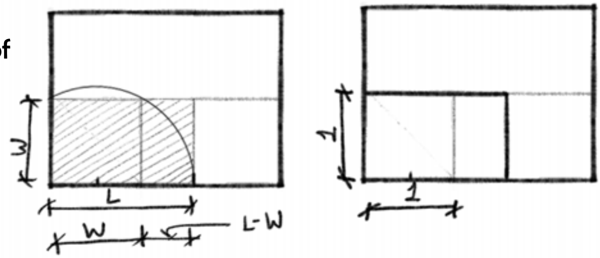


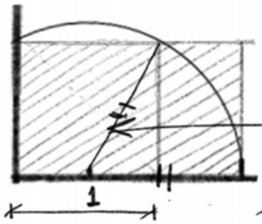
Mark the midpoint on the line of square.



Taking the distance from one side of the square to an opposite corner as radius, draw an arc.

This gives the length of the golden rectangle.





$$\sqrt{\frac{1}{2}^2 + 1^2} = \frac{\sqrt{5}}{2}$$

$$W = 1, \quad L - W = \frac{\sqrt{5}}{2} - \frac{1}{2}, \quad L = \frac{\sqrt{5}+1}{2}$$

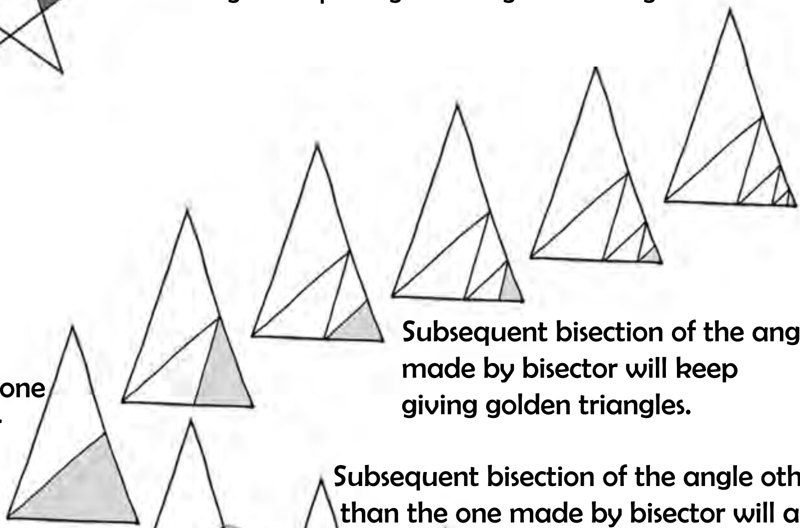
$$\frac{L}{W} = \frac{\frac{\sqrt{5}+1}{2}}{1} = 1.618$$

(Golden ratio)

A whole book has been written on this number by Mario Livio (The Golden Ratio: The Story of Phi, the World's Most Astonishing Number) "The fascination with the Golden Ratio is not confined just to mathematicians. Biologists, artists, musicians, historians, architects, psychologists, and even mystics have pondered and debated the basis of its ubiquity and appeal."

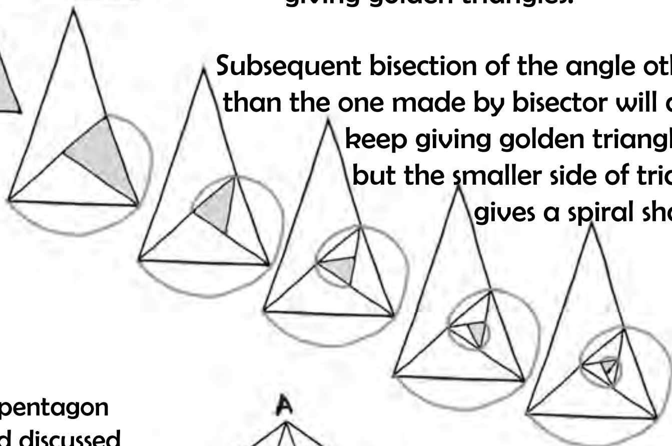
34 Golden Ratio in Triangle & Pentagon

The triangle of a pentagram is a golden triangle.



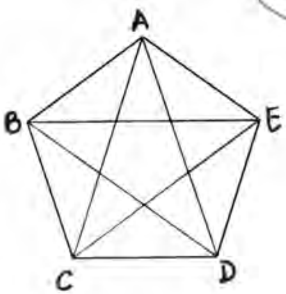
bisect one bigger angle

Subsequent bisection of the angle made by bisector will keep giving golden triangles.



Subsequent bisection of the angle other than the one made by bisector will also keep giving golden triangles, but the smaller side of triangle gives a spiral shape.

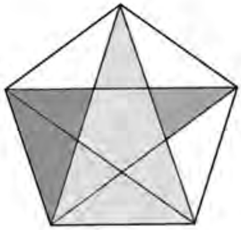
Make a regular pentagon using the method discussed in the earlier activity. If we join the diagonals of the pentagon, we get a star shaped structure inside which is called a pentagram.



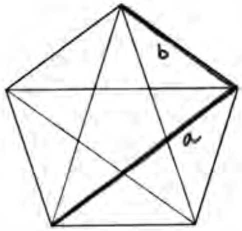
In the pentagon ABCDE, all the diagonals are in golden ratio with the sides of the pentagon.

AB and AD are in golden ratio In the same manner, AB and BE are also in golden ratio. Can you identify other sides in golden ratios?

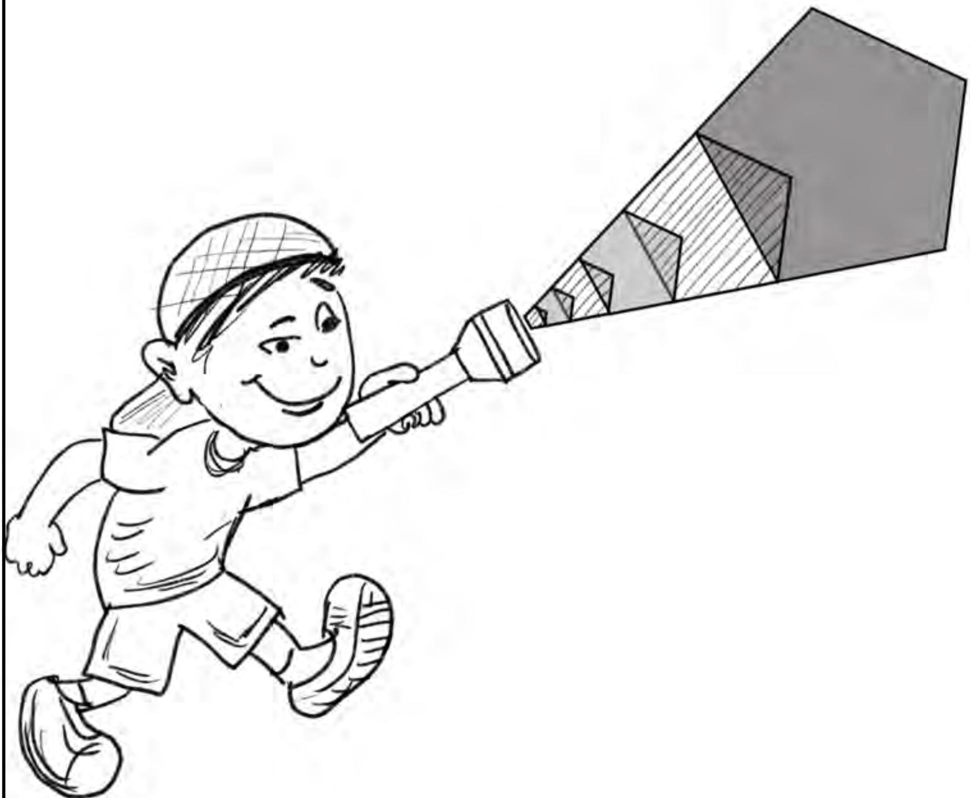
In a golden pentagon the golden ratio is in triangles formed inside pentagon.



3 shaded triangles are golden triangles

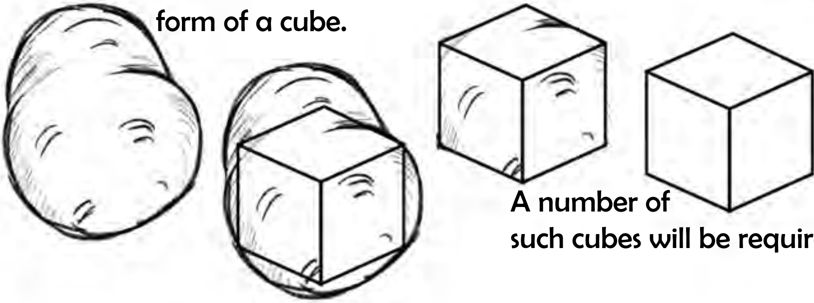


a/b or diagonal/ side is in golden ratio. So a pentagon with diagonal as previous pentagon's side will next pentagon.



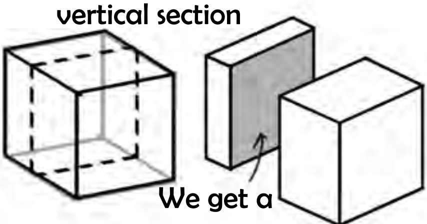
35 Polygons from Cube

Take a potato and cut it in the form of a cube.



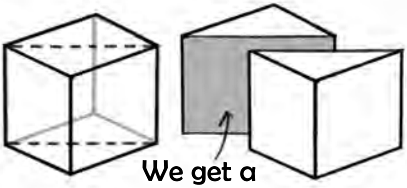
A number of such cubes will be required.

Let us cut a simple vertical section



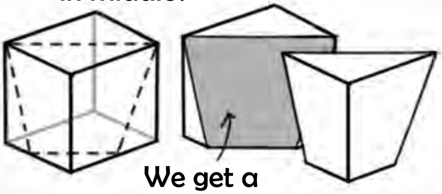
We get a square section.

Cut a diagonal section now.



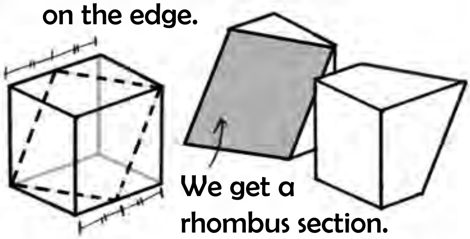
We get a rectangular section.

What will happen if we incline the diagonal plane somewhat outward so that bottom part of section cuts the two front faces in middle?



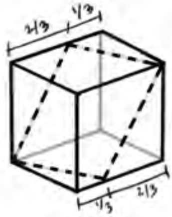
We get a trapezoidal section!

Now incline the diagonal plane such that the diagonally opposite vertices of the plane lie midway on the edge.

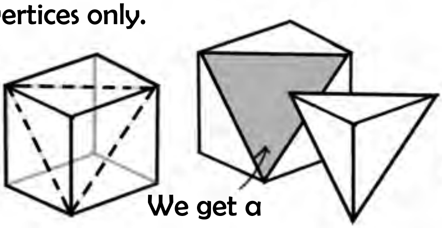


We get a rhombus section.

What will happen if the diagonally opposite vertices of the plane does not lie midway but say at 1/3rd of edge? (parallelogram)

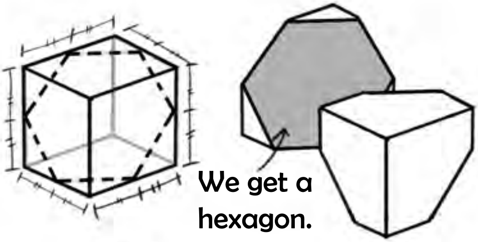


Cut diagonally such that the sectional plane touches 3 vertices only.



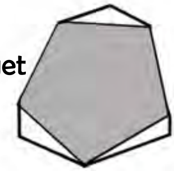
We get a triangle.

What will we get if the sectional plane touches the 6 edges at midpoint?



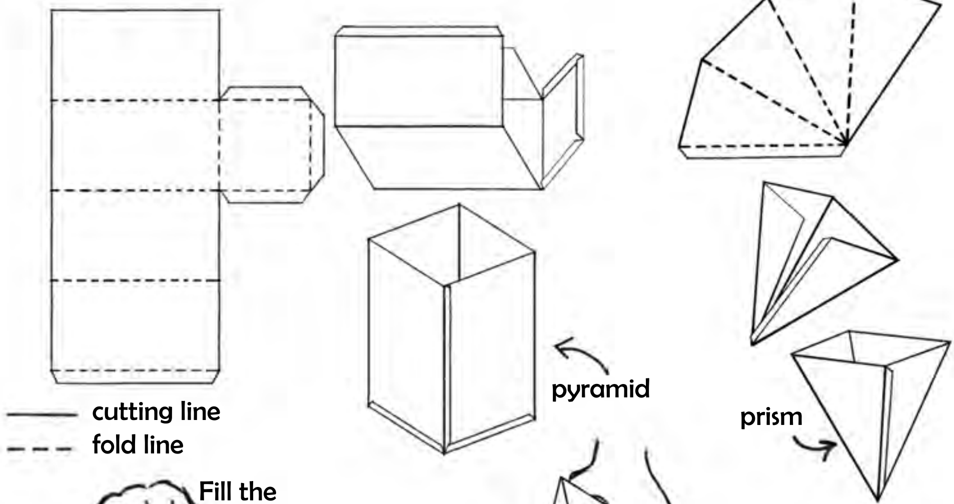
We get a hexagon.

How can we get a pentagon?



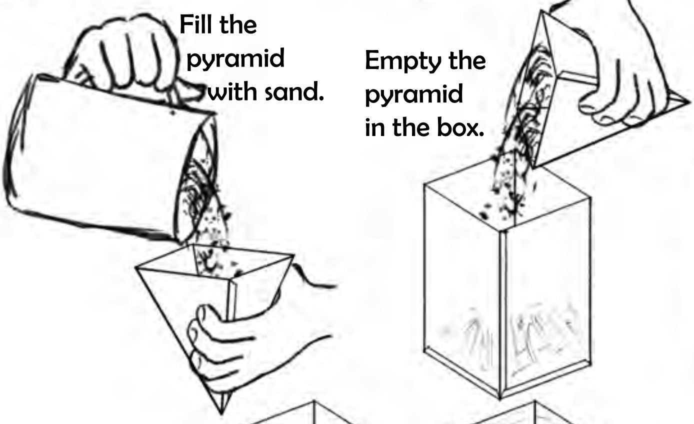
36 Volume of Pyramid is 1/3 of Prism

We will make a square prism(a cuboid) and a square pyramid and then compare the volume of the two.

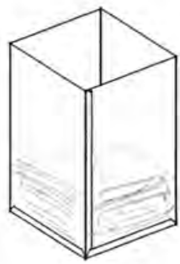


Fill the pyramid with sand.

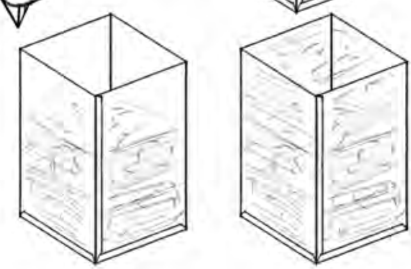
Empty the pyramid in the box.



Box will not fill completely.

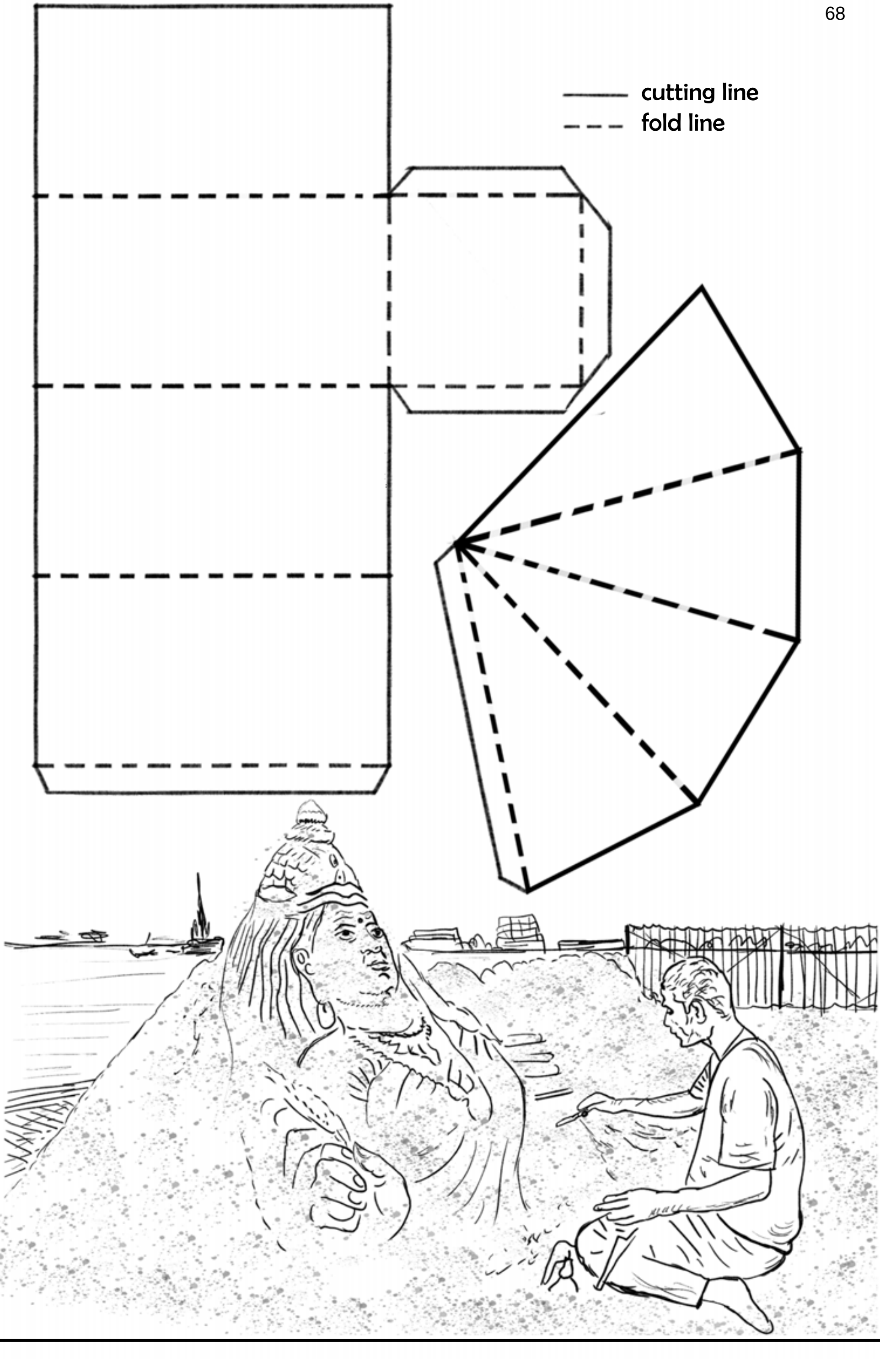


Repeat the filling and emptying.

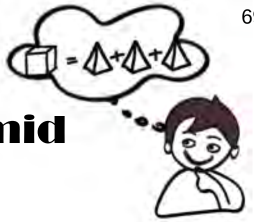


Again fill the pyramid and empty it in the box. It is completely filled the third time.

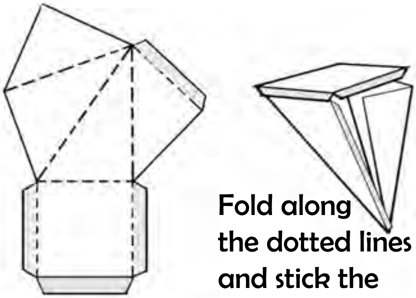
So volume of prism = 3 times volume of pyramid
or volume of pyramid = 1/3 volume of prism


cutting line
fold line

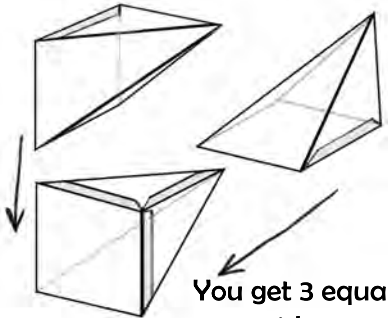
37 Volume of a Cube = 3 Equal Square Pyramid



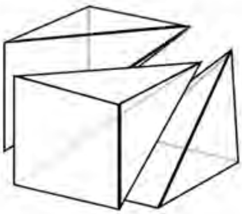
Make three identical pyramids with the help of blue print given here.



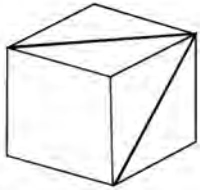
Fold along the dotted lines and stick the grey part on the faces.

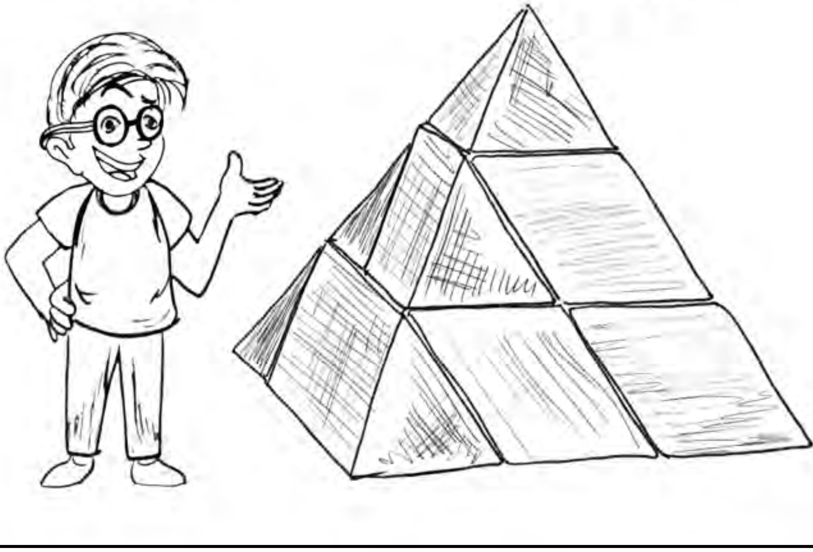


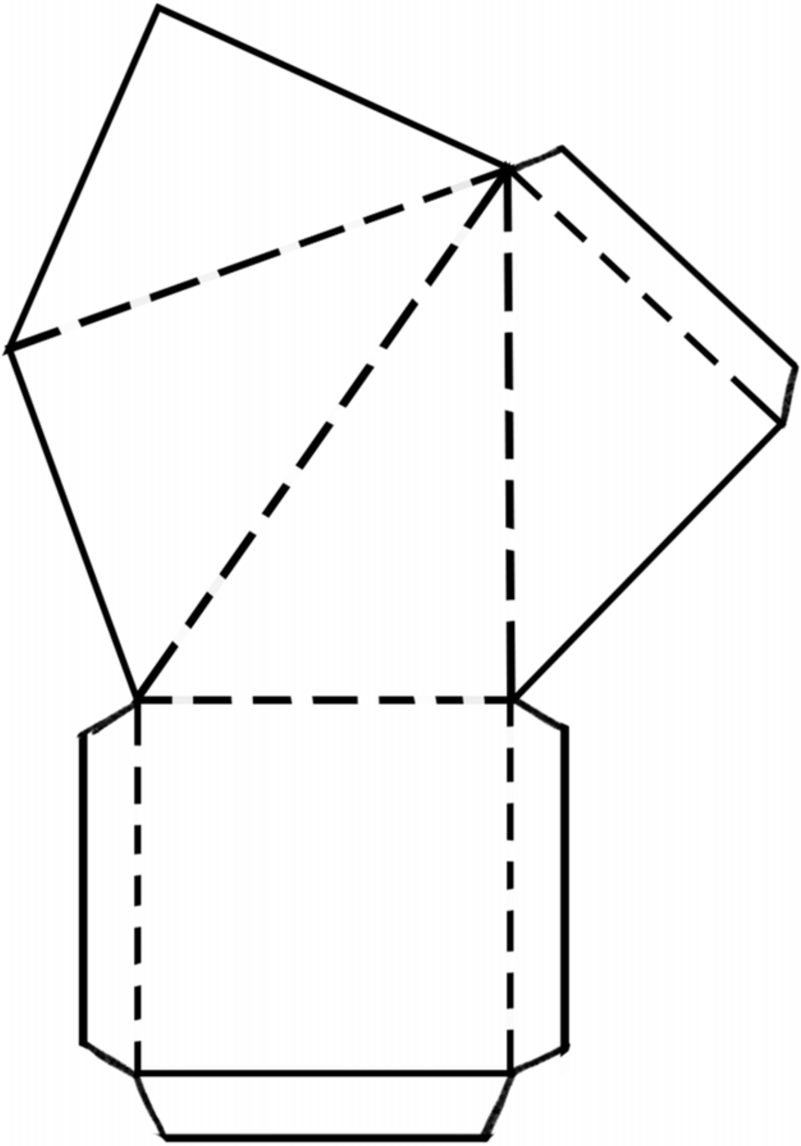
You get 3 equal pyramids



Join the 3 pyramids to get a cube



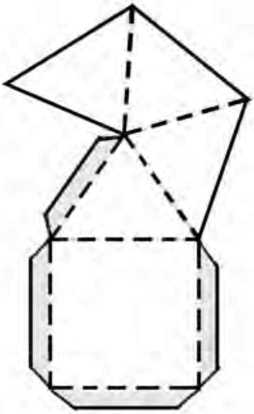


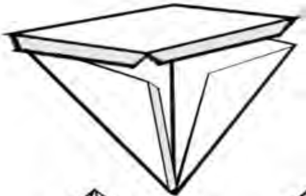


cutting line
fold line

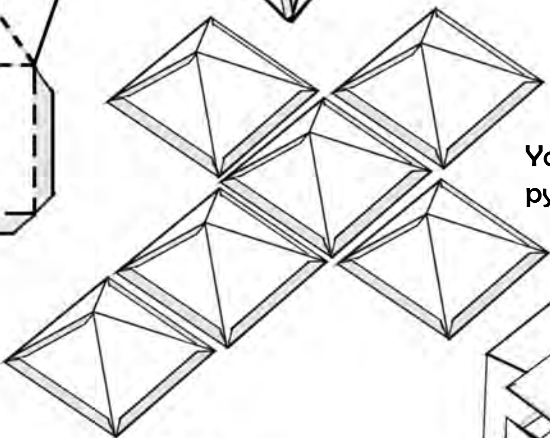
38 Volume of a Cube = 6 Half Square Pyramid

Make six identical pyramids with the help of blue print given.

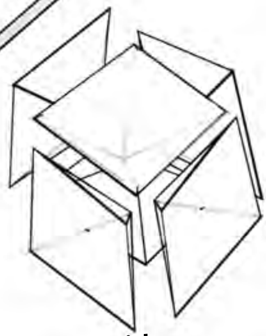




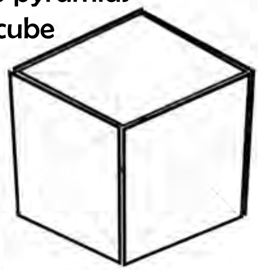
Fold along the dotted lines and stick the grey part on the faces

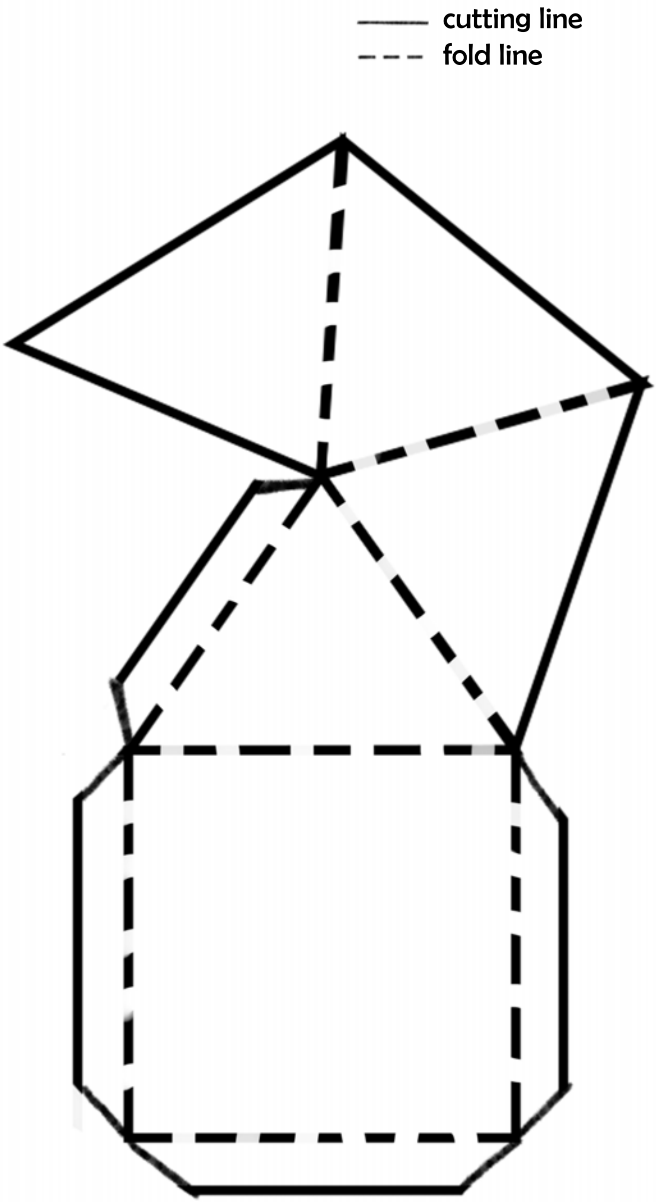


You get 6 equal pyramids



Join the 6 pyramids to get a cube

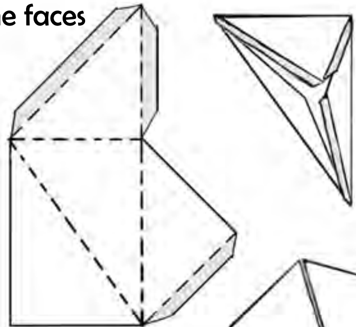




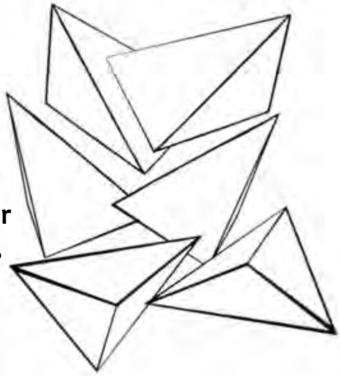
39 Volume of a Cube = 6 Triangular Pyramid

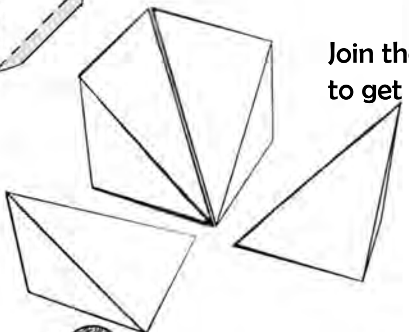
Make six identical pyramids with the help of blue print given.

Fold along the dotted lines and stick the grey part on the faces

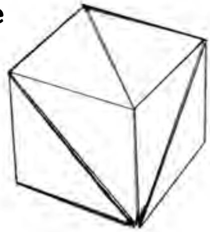


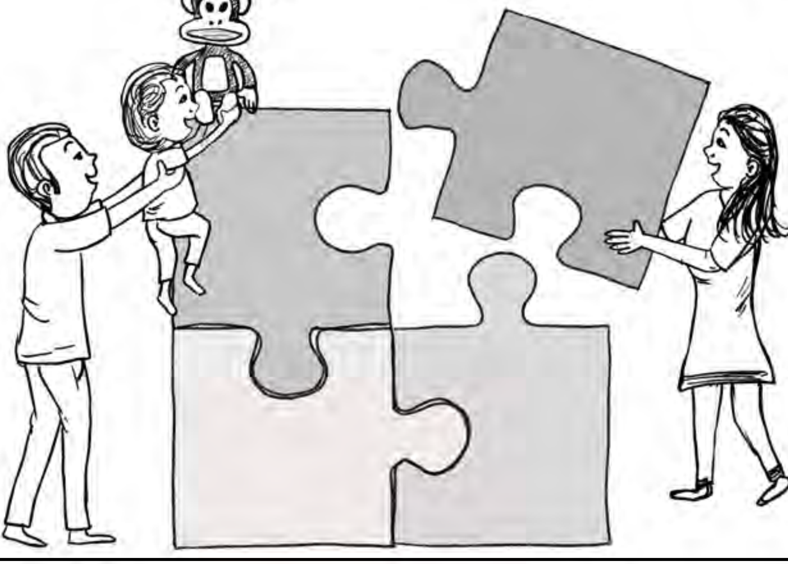
You get 6 equal triangular pyramids

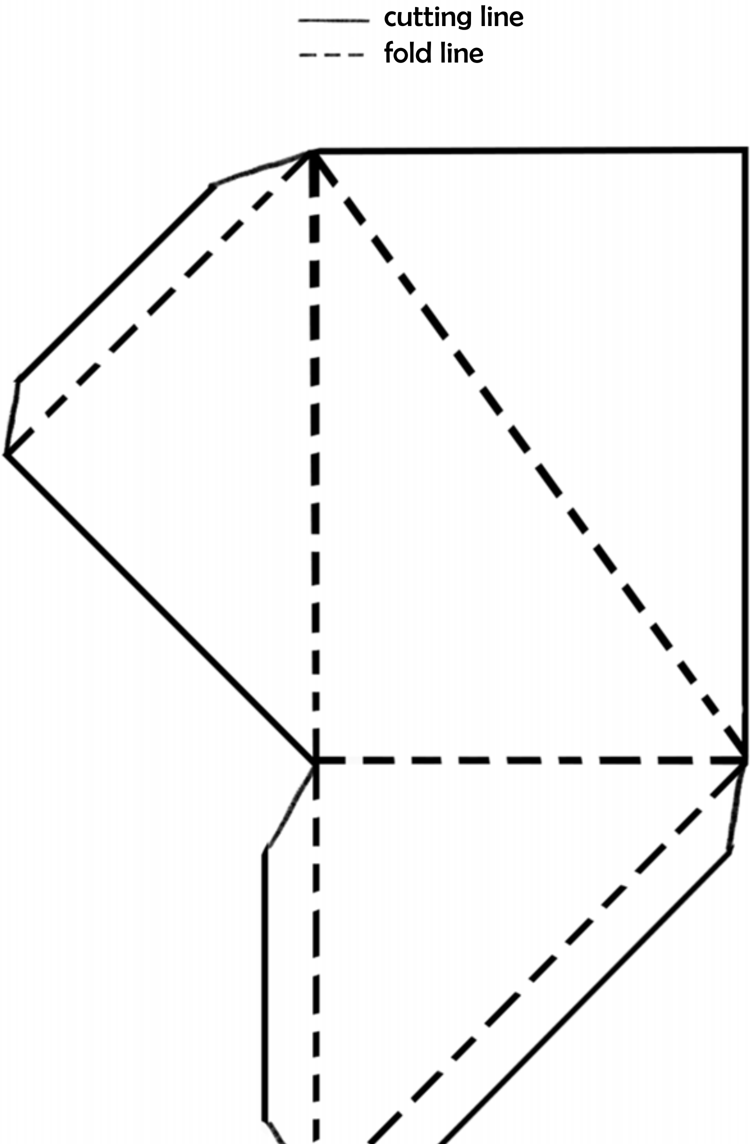




Join the 6 pyramids to get a cube

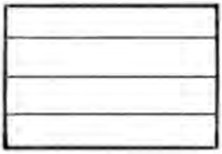


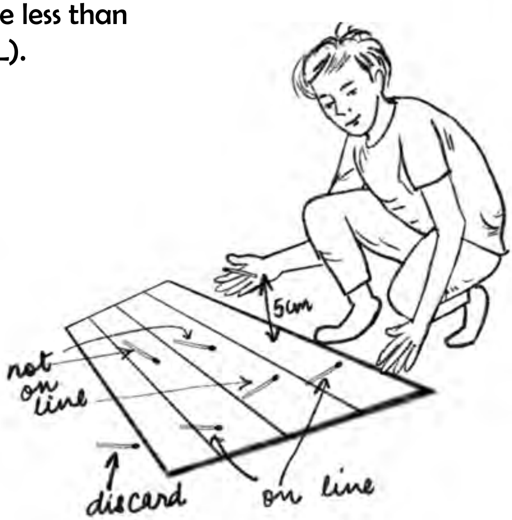



cutting line
fold line

40 Buffon's Needle

Make parallel lines on a sheet of paper. The spacing(x) of lines must be less than the length of the matchstick(L).





Place the paper on a level ground. Drop a matchstick from an approximate height of 5cm on the paper.

Observe the position of sticks. It will either touch a line or fall in between two lines without touching any line. Discard the ones which drop out of paper. Drop 20 such sticks.

Make a table

Stick position	Tally	total	propor-tion
on line	𝍸 I	6	6/20 = 0·3
between lines	𝍸 𝍸 IIII	14	14/20 = 0·7
	Total	20	

How many sticks fell on line? Calculate the proportion(p)

Now calculate the value of pi from the formula $\pi = 2L/xp$

Experiment with different number of sticks, or different spacing or different length of sticks. Make sure that the length of stick is always less than the spacing of lines. Observe.

We would see that we get more accurate value of π when we drop more number of sticks.

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