
#### Abstract

ABOUT CCL In these changing times we need to re-imagine our education. We need to make our classes engaging, provide experiential learning, foster creativity/innovation and focus on concepts. This is the only way forward for us to face the future that lies ahead. We need to get out of rote learning and provide experiences to children that they can relate to in the classes. This can be achieved by inspired teachers/facilitators, experiential learning with deep understanding of concepts.


Center for Creative Learning (CCL) at IIT Gandhinagar was set up in 2017 to bring back the gleam in the eyes of our children and teachers. The goal was to create and disseminate science/math content which will make learning engaging, joyous and conceptual. All this would be done with inexpensive material so that every child can have access. Since its inception, we have developed 200+ hands-on activities, conducted over 50 hands-on experiential workshops with 6000+ teachers. The idea of these workshops was to expose the teachers to the power of hands-on learning. The feedback from the teachers has been overwhelming.

We have worked with Kendriya Vidyalaya (KV) teachers from across the country, teachers of Chhattisgarh, Maharashtra, Ahmedabad Municipal Corporation. All the teachers unanimously stressed on the need to provide this experience to every teacher across the country. We also realize a great need to scale our workshops and reach out to all the teachers of the country. This reach is possible using technology.

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## PLATONIC SOLIDS

## LEARNINGS

3D shapes
Polyhedrons
Vertex

Edges

Faces of
Polyhedron


Named after the Greek philosopher Plato, a platonic solid is constructed by congruent (identical in shape and size) regular (all angles equal and all sides equal) polygonal faces with the same number of faces meeting at each vertex. The interesting thing about Platonic solids is that only 5 solids meet these two criteria - Tetrahedron, Hexahedron, Octahedron, Icosahedron and Dodecahedron.

## WHAT'S GOING ON?

1. It is also interesting that despite having only two con-straints- congruent regular faces with same number of faces meeting at each vertex- there are only five such solids.
2. Let's start with the simplest polygon - triangle. Join three triangles at all vertices which is the minimum number required to get a closed shape. The shape you get is a tetrahedron.
3. With four triangles at a point, you get an octahedron. With five, an icosahedron.
4. If you try to make a solid with 6 triangles at a vertex, you would find that you can't make a closed solid as the shape becomes flat ( $60^{\circ}$ of equilateral triangle $\times 6$ $=360^{\circ}$ ).
5. With more than 7 triangles, the total angle at a vertex becomes more than 360 and you can't get a convex solid.
6. So there are 3 platonic solids using triangles. Next, take the four sided regular polygon, a square.
7. With 3 squares at a point, you get a cube. Place four squares at a vertex and the shape again becomes flat $\left(90^{\circ} \times 4=360^{\circ}\right)$. With more than 4 squares at a point, you get angle more than 360 degrees.
8. Now, using 3 pentagons at a vertex, you get a dodecahedron. With 4 pentagons at a vertex, the angle becomes more than $360^{\circ}\left(108^{\circ} \times 4=432^{\circ}\right)$
9. If you use 3 hexagons at a point, the shape becomes flat again $\left(120^{\circ} \times 3=360^{\circ}\right)$.
10. And that's it. We have proved that there can only exist five!

## EXPLORE

The Platonic solids have been known since antiquity. In the 16th century, the German astronomer Johannes Kepler attempted to relate the five extraterrestrial planets known at that time to the five Platonic solids. He proposed a model of the Solar System in which the five solids were set inside one another and separated by a series of inscribed and circumscribed spheres. Kepler's original idea had to be abandoned, but out of his research came his three laws of orbital dynamics, the first of which was that the orbits of planets are ellipses rather than circles, changing the course of physics and astronomy.

## TETRAHEDRON

## LEARNINGS

3D shapes

Polyhedrons
Vertex

Edges

Faces of
Polyhedron


Tetrahedron is the simplest of all the Platonic solids. It is formed by joining three triangles at each vertex. The bond of carbon atoms (for example, in $\mathrm{CH}_{4}$ ) is arranged in a tetrahedral geometry.

## WHAT'S GOING ON?

A tetrahedron has four triangular faces (tetra - 4, hedronfaces)
Total vertices for these 4 triangles $-4 \times 3=12$.
As three triangles are joined at each vertex of tetrahedron, total vertices of tetrahedron $=12 / 3=4$.
Total edges for 4 triangles $=4 \times 3=12$.
As two triangles of tetrahedron share the same edge, total edges of tetrahedron $=12 / 2=6$.


## LEARNINGS

3D shapes
Polyhedrons
Vertex
Edges
Faces of
Polyhedron


Cube is the most famous Platonic solid which is formed by joining three squares at each vertex.

## WHAT TO DO?

Take out the pieces from the flat sheet given.
Use the given joints to and rubberbands to attach the squares at 90 degrees to compleat the cube.

## WHAT'S GOING ON?

1. A cube, also called hexahedron, has six square faces (hexa - 6, hedron- faces)
2. Total vertices for these 6 squares $-6 \times 4=24$.
3. As three square are joined at each vertex of cube, total vertices of cube $=24 / 3=8$.
4. Total edges for 6 squares $=6 \times 4=24$.
5. As two squares of cube share the same edge, total edges of cube $=24 / 2=12$.

## EXPLORE

The solid obtained by joining the center of the faces of a polyhedron is called its dual structure. If you join the center of the faces of a cube, you get a structure called octahedron which is another Platonic solid. This means that the dual of a cube is octahedron.

For any two dual structures, the number of faces and vertices is swapped, and the number of edges is same.

|  | FACES | VERTICES | EDGES |
| :--- | :---: | :---: | :---: |
| SOLID | 6 | 8 | 12 |
|  |  |  |  |
| DULE | 8 | 6 | 12 |



## OCTAHEDRON

## LEARNINGS

Vertex
Edges
Faces of
Polyhedron
Faces of
Polyhedron

3D shapes<br>Polyhedrons



Octahedron is one of the Platonic solids which is formed by joining 4 triangles at each vertex.

## WHAT'S GOING ON?

1. An octahedron has eight triangular faces (octa - 8, he-dron- faces)
2. Total vertices for these 8 triangles: $8 \times 3=24$.
3. As four triangles are joined at each vertex of octahedron, total vertices of octahedron $=24 / 4=6$.
4. Total edges for 8 triangles $=8 \times 3=24$.
5. As two triangles of octahedron share the same edge, total edges of octahedron $=24 / 2=12$

## EXPLORE

1. Can you see three squares in the octahedron in three perpendicular planes?


## DODECAHEDRON

## LEARNINGS

3D shapes
Polyhedrons

Vertex

Edges

Faces of
Polyhedron


Dodecahedron is one of the Platonic solids which is formed by joining three pentagons at each vertex.

## WHAT'S GOING ON?

A dodecahedron has four twelve pentagonal faces (dodeca - 12, hedron- faces)
Total vertices for these 12 pentagons $-12 \times 5=60$ As three pentagons are joined at each vertex of dodecahedron, total vertices of dodecahedron $=60 / 3=20$.
Total edges for 12 pentagons $=12 \times 5=60$.
As two pentagons of dodecahedron share the same edge, total edges of dodecahedron $=60 / 2=30$.


## ICOSAHEDRON

## LEARNINGS

3D shapes
Polyhedrons
Vertex
Edges
Faces of
Polyhedron


Icosahedron is one of the Platonic solids which is formed by joining five triangles at each vertex. It is one of the most beautiful Platonic solids, which resembles a ball. Icosahedron is the structure on which the Geodesic domes (popularized by Buckminster Fuller) are based.

## WHAT'S GOING ON?

1. An icosahedron has twenty triangular faces (icosa - 20, he-dron- faces)
2. Total vertices for these 20 triangles: $20 \times 3=60$.
3. As five triangles are joined at each vertex of icosahedron, total vertices of octahedron $=60 / 5=12$.
4. Total edges for 20 triangles $=20 \times 3=60$.
5. As two triangles of icosahedron share the same edge, total edges of icosahedron $=60 / 2=30$

## EXPLORE

The solid obtained by joining the center of the faces of a polyhedron is called its dual structure. If you join the center of the faces of an icosahedron, you get a structure called dodecahedron which is another Platonic solid. This means that the dual of an icosahedron is dodecahedron.

For any two dual structures, the number of faces and vertices is swapped, and the number of edges is same.

|  | FACES | VERTICES | EDGES |
| :--- | :---: | :---: | :---: |
| SOLID | 20 | 12 | 30 |
|  |  |  |  |
| DUEL | 12 | 20 | 30 |




## CIRCLING AROUND A GEAR

## LEARNINGS

Gear Ratio


If you rotate a coin around a fixed coin of the same diameter, how many times would it rotate to cover the perimeter once? Rotate the gears and get ready to be amazed!

## WHAT TO DO?

1. Connect two gears such that the first gear is fixed and the second gear revolves around it.
2. Keep the first gear fixed and rotate the second gear around the first gear. Observe the number of rotation the second has complete on its own axis when it completes one rotation around the first gear.
3. Now, connect a third gear with the second gear.
4. Observe the number of rotations of the third gear.

## WHAT'S GOING ON?

1. The first gear is fixed at one place and the second and third gears revolve around it.
2. If the second gear is half in size as compared to the first gear, the number of teeth in second gear are half of that in first gear.
3. If these two gears were fixed at one place, the second gear would have rotated twice as much as the first gear.
4. The second gear is also going around the first gear. As the direction of rotation and revolution is same, the total rotations of the second gear in a single journey around the first gear: $2+1=3$.
5. This means that the second gear rotates three times in one revolution around the first gear.
6. The third gear also goes around the first gear. In this case, the direction of rotation and revolution is opposite. Therefore, total number of rotations: $-2+1=-1$.
7. This means that the third gear rotates one time in one revolution around the first gear, and in opposite direction.
8. If the size of first and third gears becomes same, the third gear doesn't rotate at all $(-1+1=0)$.

## EXPLORE

What happens if you add a fourth, fifth gear and so on. Can you generalize this for n gears?





## MECHANICAL FROG

## LEARNINGS

Walking
Machine
Distance
Speed
Time

## DC Motor

Circular to
Linear Motion
Dynamic
Mechanism


A DC motor and some MDF parts is what it takes to get this mechanical frog hopping and the maker hopping right behind it, treating it almost like a pet.

## WHAT'S GOING ON?

1. With every rotation of the motor, the frog moves forward for a small distance.
2. When the frog-shaped pieces touch the ground, they act like an anchor and the body of the frog moves forward.
3. In the next cycle, the body touches the ground and the frog-shaped pieces are displaced forward.
4. This cycle repeats and the frog keeps hopping forward!

## EXPLORE

How much does each leg move in one step (in terms of the radius of the gear)?

Organize a race for different frogs (you can also make other frogs by cutting card board in shapes given in the MDF sheet.)




## SWIMMING FROG

LEARNINGS<br>Mechanics

Gears


The frog looks like swimming when you roll the wheels forward. It is a fascinating toy which is really simple to build. The hands and legs are attached to the wheels and move back and forth when the wheels rotate.

## WHAT'S GOING ON?

The legs of the frog are connected to the rear wheel such that when you push the frog forward, the legs make a linear motion. They move forward and backward, mimicking the swimming movement of legs. So circular motion is converted into linear motion. The legs, in turn, are connected to the hands which also move when you move the frog. And the frog looks like swimming on the floor.

## EXPLORE

Make similar toys using cardboard. Cut out the pieces and make different motions.







## UP IT UP



Spatial
Cognition
This is an beautiful puzzle with 8 cubes. The aim of the puzzle is to invert all the cubes upside down by just rolling them. It was discussed in Martin Gardner's Mathematical Games column in the Scientific American in 1975.

## WHAT TO DO?

The puzzle consists of eight identical cubes in a $3 \times 3$ grid with an empty middle cell. You can roll a cube at a time into the empty space. A move consists in rolling a cube to an empty cell and the aim is to invert all eight cubes upside down. In how many moves you can solve this puzzle?

## WHAT'S GOING ON?

1. The creator of the puzzle, John Harris, has reported a 38 -move solution to this puzzle.
2. In reference to the empty cell, the cubes are named up, down left and right ( $\mathrm{U}, \mathrm{D}, \mathrm{L}, \mathrm{R}$ ).
3. Roll the cubes according to the sequence given below: URDL, DRUL, LDRR, UULD, RUL; LDR, ULDD, RRUL, LDRU, LURD.

## EXPLORE

1. Each cube has 6 sides. Therefore eight cubes have $6^{8}$ $=1,679,616$ possible orientations. In the $3 \times 3$ grid, possible number of positions will be $9 \times 6^{8}=15,116,544$. This is the possible number of orientations you can have with the 8 cubes!
2. Try to find a solution having moves lesser than the given solution (of 38 moves).



## TILT STICK

## LEARNINGS

Geometry

Trigonometry


Can a straight stick pass through a small curved slot? In this activity, you can make a rotating mechanism which lets the straight stick pass through a hyperbolic slot. The stick is tilted according to the curvature of the hyperbola. It is extremely satisfying to watch the stick pass through!

## WHAT'S GOING ON?

The stick passes through the hole, but not all at the same time. First the bottom part goes in, and then gradually the entire stick passes through the hole. If the entire stick had to pass through at the same time, the hole had to be a straight line. But as different parts of the stick pass at different times, the hole is a curved slot.

## EXPLORE

1. The projection of a line on a plane is generally a line, and not a curve. Because the stick is slightly tilted and generates a shape called hyperboloid while rotating. The projection of this hyperboloid on a plane is a hyperbola. Therefore the hole you see is in the shape of a hyperbola.
2. Although hyperboloid is a (doubly) curved surface, it can be made entirely with the help of straight lines.


## HYPERBOLOID








## TRAMMEL 4

## LEARNINGS

Ellipse

Four Bar
Mechanism


You can draw an ellipse using this trammel of Archimedes. The two shuttles move back and forth in perpendicular channels and the end point of handle moves in an ellipse.

## WHAT'S GOING ON?

1. A trammel is a mechanism that generates the shape of an ellipse. Archimedes developed this great device 2000 years back to draw an ellipse.
2. It consists of two shuttles which are confined ("trammeled") to channels (or rails) and a rod which is attached to the shuttles at fixed positions along the rod.
3. As the shuttles move back and forth, each along its channel, the end of the rod moves in an elliptical path.
4. The handle goes round and round. The pieces look like they would crash into each other but they never do.
5. It is also similar to the motion of a 4-stroke engine and crankshaft.

## EXPLORE

1. You can change the shape of the ellipse (called the eccentricity) by moving the point where the pen is attached. Can you find out the point where the pen would draw a circle?
2. You can also prove mathematically that the locus of the pen is an ellipse.
$\mathrm{x}=(\mathrm{p}+\mathrm{q}) \cos \Theta$
$y=q \sin \Theta$
Therefore, $x^{2} /(p+q)^{2}+y^{2} / q^{2}=1$
3. It is also an example of a lever that switches back and forth from a first class lever to a second class lever.





## TRAMMEL 6

## LEARNINGS

Ellipse

Six Bar
Mechanism


This is another version of the trammel of Archimedes. It has three shuttles instead of two which move in different channels to produce an ellipse.

## WHAT'S GOING ON?

1. A trammel is a mechanism that generates the shape of an ellipse. Archimedes developed this great device 2000 years back to draw an ellipse.
2. It consists of three shuttles which are confined ("trammeled") to channels (or rails) and a rod which is attached to the shuttles at fixed positions along the rod.
3. As the shuttles move back and forth, each along its channel, the end of the rod moves in an elliptical path.
4. The handle goes round and round. The pieces look like they would crash into each other but they never do. It is also similar to the motion of a 6-stroke engine and crankshaft.

## EXPLORE

1. The shuttles move along three axes. There is a major axis and other two are at offset of $120^{\circ}$. Can you identify which one is the major axis?
2. It is also an example of a lever that switches back and forth from a first class lever to a second class lever.




## 30 BALL (RHOMBIC TRIACONTAHEDRON)

LEARNINGS<br>Polyhedrons<br>3D Geometry<br>Dihedral Angle



The rhombic triacontahedron is the most common thir-ty-faced polyhedron. Therefore it is sometimes called the 30 Ball. All the 30 faces of this ball are rhombuses. Make this ball using the sheet and study all the properties. Then insert a light bulb and hang it as lamp!

## WHAT'S GOING ON?

1. This structure can be made starting from one of the platonic solids, dodecahedron.
2. Make pyramids on the pentagonal faces of dodecahedron and adjust the height of pyramids such that the two adjacent triangles come in same plane (total number of triangles formed $=12 \times 5=60$ )
3. The two triangles, combined together, make a rhombus (number of rhombuses $=60 / 2=30$ ). The resulting structure is the Rhombic Triacontahedron.
4. It is a polyhedron with 30 rhombic faces, 60 edges and 32 vertices of two types.
5. The two diagonals of the rhombus are in golden ratio, $\varphi$. Therefore the rhombus is called a golden rhombus.

## EXPLORE

1. The dihedral angle is the angle between the faces of the polyhedron. It is the angle at which the joints to join the rhombuses are cut. In this case, the dihedral angle between the adjacent faces is $144^{\circ}$. Can you get to this dihedral angle using geometry?
2. Find the height of the pyramids if the sides of the rhombus is taken as 1 unit length.



## LEARNINGS

Walking
Machine
Distance
Time and Speed
DC Motor
Mechatronics


Circular to
Linear Motion
Dynamic
Mechanism
Using only a simple DC motor and some gears, your robot is ready to walk like a human! The name is taken from the famous Rajnikanth movie "Robot" in which he played a robot named Chitti!

## WHAT'S GOING ON?

The two legs of the robot move alternatively and the body of the robot moves forward. The motor used in the robot is a dual shaft motor, which means that the shaft is coming out of both the ends of the motor. The two ends of the shaft are attached to the two gears rotating different legs. The alternate movement of the two legs is because of phase difference (of 180 degrees) in the gears that move different legs. When one leg touches the ground, that end of the shaft can't rotate further. Therefore the other gear rotates and takes one step forward. Then this rotating gear touches the ground, acts as a pivot and the other gear moves and so on. This way, the entire robot moves forward.

## EXPLORE

The two legs of the robot move alternatively and the body of the robot moves forward. The motor used in the robot is a dual shaft motor, which means that the shaft is coming out of both the ends of the motor. The two ends of the shaft are attached to the two gears rotating different legs. The alternate movement of the two legs is because of phase difference ( of 180 degrees) in the gears that move different legs. When one leg touches the ground, that end of the shaft can't rotate further. Therefore the other gear rotates and takes one step forward. Then this rotating gear touches the ground, acts as a pivot and the other gear moves and so on. This way, the entire robot moves forward.







## ABC OF MATH (ANY BASE COUNTING)

## LEARNINGS

Number
Systems

Decimal
System

Binary System

Basic Mathematical Operations


We are all comfortable in the decimal system but understanding other number systems (such as binary) really enriches our understanding about counting and other basic mathematical operations such as addition, multiplication etc.

Start counting in any number system using this activity and some beads.

## WHAT'S GOING ON?

1. Adjust the length of the base sticks such that it can fit the number of beads required by the number system. For example, if you are counting in decimal system (base 10), you should have stick length corresponding to 9 beads. For base 5 counting, the length should be adjusted to fit 4 beads.
2. Let's discuss how to count in base 5. But it can be easily extended to any base.
3. Let's count to 17 in base 5. Start putting beads in the sticks, starting from the first stick. For zero, there are no beads in any stick.
4. For one, put one bead and so on till 4 . Now the first stick is full.
5. For the next number, empty all the beads of the first stick, and put one bead in the second stick.
6. So every bead in the second stick is equivalent to 5 in this system. Try to relate it with the decimal counting we are familiar with.
7. When you reach 17 , you would see that there are 3 beads in the 2 nd stick and 2 beads in the first stick. Verify this with your result. Therefore, 17 in base 5 would be written as 32 .

## EXPLORE

1. How much can you count on five sticks if you are counting in binary system?
2. Try to add, subtract, multiply and divide numbers in other number systems.



## BRACHISTOCHRONE CURVE

## LEARNINGS

Curve of
Fastest Descent

Cycloid

Gravity


If you want to slide from point $A$ to point $B$, what should be the shape of the slide so that you reach your destination in the shortest time? At first look, it seems that the slide should be a straight line but you are in for surprising ride!

## WHAT'S GOING ON?

1. Mark a point on the circumference of a circular wheel. The path traced by the point as the wheel rolls along a straight line is called a cycloid curve.
2. The Brachistochrone (Greek word meaning 'shortest time') curve is part of the cycloid curve which is the curve of fastest descent if the object is just being pulled by gravity.
3. You want the path to be short (like a straight line) for the shortest time but you also want the object to go fast which requires a steeper slope. But increasing the steepness at the start also increases the total distance.
4. So the curve of fastest descent has to balance length with the steepness. And that's what the Brachistochrone curve does.
5. The problem was first posed by Johann Bernoulli in 1696 to the most brilliant mathematicians in the world (in particular, he was very concerned to show off that he was smarter than his brother and rival, Jacob Bernoulli).

## EXPLORE

1. The Brachistochrone curve is the same at Tautochrone (Tauto - same, chrone- time) curve which is also the cycloid. Release two marbles on this surve from anywhere and they will reach the bottom at the same time
2. The Brachistochrone curve is the fastest curve of descent for any two points on the curve, not just the highest and lowest points. It means that the object may do down and then again up on a cycloid but it would still be the fastest descent.
3. Johann Bernoulli also mailed this problem to Issac Newton who stayed up all night to solve it and mailed the solution anonymously by the next post. Upon reading the solution, Bernoulli immediately recognized its author, exclaiming that he "recognizes a lion from his claw mark". This story gives some idea of Newton's power, since Johann Bernoulli took two weeks to solve it.




## GEAR JACK

## LEARNINGS

Mechanical
Advantage


This is an example of deployable mechanism used to lift objects. A small rotation of the gear can lift the object really high!

## WHAT'S GOING ON?

1. In a normal car jack, you have to spin the screw for a long time and then jack moves up a little distance. The jack therefore provides mechanical advantage and you are able to lift the car easily.
2. In this deployable structure, a small rotation of the gear results in a large movement of object at the top - due to the scissor-like structures. The more scissors you add at the top, the more distance is covered in a single rotation of the handle.
3. So what's the catch? Can you add as many scissors as you want? No. The more the number of scissors, the more effort required to rotate the handle!
4. Therefore, this machine can be said to have negative mechanical advantage. This is also used in various real-life applications where you want to increase the amount of movement/rotation, for example, the hand blender. You have to move the handle for a small distance and the blender rotates for a long time.
5. But always remember that if the distance is increased, you'll have to exert a large amount of force (for example, a gear cycle in a higher gear). There is really no such thing as a free lunch!











## HAMILTONIAN PATH

## LEARNINGS

Geometry

Logical
Reasoning
Creative
Thinking


Can you visit the 12 vertices on the three rectangles such that you don't go on any vertex twice and the ending point is the same as starting point? This puzzle is called the Hamiltonian path.

## WHAT'S GOING ON?

1. A path that visits each vertex exactly once is called a Hamiltonian path.
2. The corners of three interlocking golden rectangles (the length and breadth are in golden ratio, 1.618) form the vertices of an icosahedron, one of the five Platonic solids.
3. Icosahedron has five triangular faces meeting at each vertex.
4. All the five platonic solids have Hamiltonian cycle connecting their vertices.
5. Here is an illustration of Hamiltonian cycle in the simplest Platonic solid - tetrahedron. Figure 1(b) shows a Hamiltonian cycle that visits all the vertices without repetition of any edges.
6. Here, the cycle is shown for the given icosahedron.


## EXPLORE

1. Explore Hamiltonian path for other Platonic solids.
2. Can you figure out a path in icosahedron which covers all the edges exactly once? Note that in the Hamiltonian cycle, you covered all the vertices, not the edges. An Eulerian path is a trail that visits every edge exactly once (allowing revisiting of vertices).
3. Among all the Platonic solids, only Octahedron can have Eulerian path. While solving the famous Seven Bridges of Königsberg problem, Euler showed a graph can have an Eulerian path only if all the vertices in the graph have an even degree (which means the number of edges connected at a vertex). In Icosahedron, 5 edges meets at all the 12 vertices, hence it can't have Eulerian path.






## LARGEST BOX FROM A RECTANGLE

## LEARNINGS

Geometry

Area of a
Rectangle

Volume of a Cuboid

Introduction to
Calculus


Suppose you have an A4 sheet and you want to make a box (a cuboid) out of it. You can do so by cutting squares from the four corners and folding up the remaining sides to make a box. The question is, how much square should you cut to get the box with the maximum volume?

## WHAT TO DO?

Fill different boxes with sand and find out which box can take maximum amount of sand. That box that holds maximum sand has the maximum volume.

## OBSERVATION

1. You would see that the area of the base gradually decreases and the height of the box increases.
2. Initially the base area is more but height is very small (*photo of box 1*). Therefore the box has very small volume.
3. In the innermost box (*photo of box $4^{*}$ ), the height is maximum but the base area is very small. Therefore this box also won't hold large amount of sand (and hence has low volume).
4. The height has to be somewhere in between if you want to maximize the volume.
5. You would find that the box 2 has the largest volume. (*photo of box 2*)

## WHAT'S GOING ON?

1. The box marked as 2 corresponds to the maximum volume.
2. For an $A 4$ sheet ( $21 \mathrm{~cm} \times 29.7 \mathrm{~cm}$ ), you have to cut the square of 4 cm ( 4.04 to be exact) to get the box with maximum volume. You can check the length of this square on the sheet and verify if it is 4 cm .

## EXPLORE

1. Are there some boxes which have the same volume?
2. This problem of maximum volume can also be solved accurately by writing the volume in terms of the length of the square and then 'differentiating' it. This branch of mathematics is called calculus and helps us find when the things are maximum or minimum.



## REAULEUX CAR

## LEARNINGS

Curve of
Constant Width
Reauleux
Triangle


This carwith a non-circularwheel gives a surprisingly smooth ride. Unlike circular wheels, these wheels don't have a set radius but they do have an unchanging diameter. Explore the magical property of the Reauleux triangle in this activity.

## WHAT'S GOING ON?

1. If you put a book on a triangle (or a square) and rotate it, the book would move up and down. What should you use to keep the book at the same level? The first choice is a circle. But can you think of any other shape which would be of constant width?
2. The car has two different types of wheels. One is circular, whereas another is formed by rounding the sides of an equilateral triangle, as shown in the figure 1(a). This shape is called the Reauleux triangle.
3. Reauleux triangle is called a curve of constant width. It means that for every pair of parallel lines touching the shape without crossing it, it will always have the same distance from each other, as shown in figure 1 b .
4. Add another pair of supporting parallel lines to the old ones and we obtain a square. The wheel performs a complete rotation while staying within the square and at all times touching all four sides of the square (figure $1 \mathrm{c})$.
5. TheshaftisalsoconnectedtoanothersmallerReauleuxtriangle which rotates inside a square. Due to constant width property, the top of the car smoothly moves at same horizontal level, without up or down movement (Figure 1d).

## EXPLORE

1. Take a circular wheel of diameter 7 cm and a Reauleux triangle, also of width 9 cm . Compare the perimeters of both shapes.
2. Can you create the constant width shape from the regular pentagon? Is it necessary that the pentagon should be regular?
3. People have made real bicycles with Reauleux traingles as wheels, literally reinventing the wheel!
4. There are also solids of constant width. The obvious one is the sphere but if you make a solid by rotating the Reauleux triangle, the resulting solid would also of constant width. You can also make the solid of constant width starting from a tetrahedron (Meissner tetrahedron).
5. The area of any solid of constant width is $\pi x$ (height)



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## MONTY HALL PUZZLE

## LEARNINGS

Statistics

Probability


Suppose you're on a game show, and you're given the choice of three doors: behind one door is a car; behind the others, goats. You pick a door, say Door 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to stay with Door 1 or do you want to switch to Door 2?" Is it to your advantage to switch? The kit contains 3 doors (with hinges) which can be used to play the game and understand the probability of the event. This game and the discussion that follows is hugely popular and counter-intuitive.

## WHAT TO DO?

1. Take out the pieces from the sheets and build the three doors.
2. Act as the game host and one other person can be the participant. Place a small car behind one door and toy goats
behind other two. After the arrangement, you must remember which gate has the car behind it and which has a goat.
3. Ask the participant to choose a door. Now open another door (except the one chosen by the participant) which has the goat behind it.
4. Ask the participant whether he wants to stay or switch to another door?
5. Repeat the game $10-20$ times and note down the outcome each time.
6. Surprisingly, you would find that the participant who switches every time wins more games!

## WHAT'S GOING ON?

1. For the initial choice the participant makes, the probability of winning the car is $1 / 3$.
2. The probability that the car is behind one of the remaining two doors is $2 / 3$. Now the host, Monty, filters the remaining doors and opens the one having the goat behind it.
3. So the probability that the car is behind the last door is $2 / 3$. So we are twice as probable to win if we switch!
4. The Monty Hall problem is popular precisely because it is both simple and counterintuitive at the same time. It is named after the host of the American television show host Let's Make a Deal where the participants of the show were offered this choice.
5. At first, it seems that after Monty has opened one door that has the goat behind it, it doesn't matter, probabil-ity-wise, whether you stick to your original choice or switch to the other door. The probability of winning seems equal in both cases. But in reality, it is always better for the participants to switch, if they want to win the car (If somebody wants the goat, that's a different story!).
6. You can think about it this way. Monty filters the doors for you. He always opens the door having the goat behind it. If you had 100 doors to choose from and only one door has the car behind it, the chances of winning are pretty slim $(1 / 100)$. But once you choose one door and the host opens all the doors having goats behind them, now it seems intuitive that switching is the better option (you would win 99/100 times)






## LEVER SYSTEM

## LEARNINGS

Laws of Lever


Give me a place to stand, and I will move the world, said Archimedes 2000 years ago. The Greek mathematician, scientist, and inventor was discussing the principle of lever. With this model, you may not be able to move the Earth but you can surely understand the underlying ratios of force, load, and distance using this simple machine.

## WHAT TO DO?

1. Place a unit weight at 5 cm mark.
2. Now balance this weight by placing a different weight on the other side.
3. To balance weights of different amounts on both sides, how should we place them? Should the heavier load be closer to the fulcrum or the lighter load? How much? Explore all this using the model.

## WHAT'S GOING ON?

1. A lever amplifies an input force to provide a greater output force.
2. The ratio of the output force to the input force is called the mechanical advantage of the lever.
3. The effort arm is equal to the distance from the fulcrum to the point of applied effort, and the load arm is equal to the distance from the fulcrum to the load weight (which is to be lifted).
4. Effort (force applied by you) multiplied by the length of the effort arm is equal to the load multiplied by the length of the load arm.
5. This means that the longer the effort end, the less the force required to raise the load.
6. Therefore, if you are trying to lift a particularly heavy stone, it is best to use a longer stick.

## EXPLORE

1. If two children are playing on a seesaw, and their weights are different, should the heavier child sit near the fulcrum or away from it, to balance the seesaw?
2. Take a pipe and hang a bag on it. Now lift the bag by holding the pipe close to the bag. Guess the weight of the bag. Now try lifting the same bag by gradually moving the hand away from the bag. Does the bag feel heavier? Why? The weight of the bag has surely not changeed. So why do you need to apply a larger force to lift the same bag?




## OPTICAL BENCH

## LEARNINGS

Convex Lens

Image
Formation


We often use projectors but seldom know how they work. This optical bench will help you experiment with the process of image formation, and understanding the concepts of real and virtual images, convex lenses and their focal lengths.

## OBSERVATION

Examine the slide projector, and use it to produce an image. Draw a ray diagram showing how the projector produces an image. What sort of an image is this? How can you tell? Now use the magnifying glass to produce an image. Draw a ray diagram to show how this image is formed. What sort of image is this one?

## WHAT'S GOING ON?

An optical bench typically consists of a long, rigid member with a linear scale applied to it. Holders for light sources, lenses and screens are placed on the apparatus so that image formation can be observed. In order to replicate the working of a projector, it must be ensured that the image produced is real (otherwise you wouldn't be able to project it onto a screen) inverted and magnified. Thus, the object is placed upside down and a convex lens is used for the entire setup - exemplifying exactly what happens when the object is located between F and 2F of convex lens.

## EXPLORE

Adjust the object and lens such that you can determine the focal length of the given lens. Move around the elements of the setup and see where it leads.

Human eye. the image is inverted again in the mind.








## PERISCOPE

## LEARNINGS

Optics
Reflection
Plane mirror


Periscopes have proven to be of great help to submariners by allowing these professions to have a view above the surface of water and aid in navigation underwater. Make your own periscope and look beyond the wall!

## WHAT'S GOING ON?

1. A periscope is an optical instrument that allows objects that are not in direct line of sight to be viewed.
2. It works on the law of reflection. The light from the object falls on one mirror and is reflected.
3. The reflected light then falls on another mirror and is again reflected and finally reaches the human eye
4. Therefore, the periscope is used to see objects which are directly not in sight. For example, if you are near a high wall, you can't see what's happening on the other side of the wall. But you can easily see beyond the wall using the periscope.




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## SINE WAVE CAR

## LEARNINGS

Trigonometry

Definition of Sine Curve

Length of an arc


Allow the textbook trigonometry come to life when the pen draws a sine curve with the movement of the car. By adapting the mechanism appropriately, the frequency and amplitude of the curve can be changed.

This car is the sine qua non of teaching and learning trigonometry!

## WHAT'S GOING ON?

1. The gear fixed on the axle of the car rotates with the wheels of the car in the same vertical plane.
2. Another gear is joined to this gear in the horizontal plane.
3. A circular disc is attached with this gear on the top. Therefore, this disc rotates in the horizontal plane when the car moves in any direction.
4. The stick on the disc moves the module on the top (on which the pen is attached) in a straight line.
5. So the car moves forward and the pen moves in a perpendicular direction to it. So theoretically, the car could make any curve. So why does it make the sine curve?
6. Because it is the definition of a sine curve. When the wheel of the car moves $\theta$, the car moves $r \theta$ ( $r$ is the radius of the wheel) in the forward direction. And the module shifts $\mathrm{R} \sin \theta$ ( R is the radius of the top disc) in the perpendicular direction to the motion of the car
7. The pen, therefore, draws the graph of $R \sin \theta$ vs $r \theta$ which is a sine curve. $(y=R \sin \theta ; x=r \theta$ $y=R . \sin (x / r)$ which is the equation of a sine curve.

## EXPLORE

1. Try changing the amplitude and frequency of the curve. What effect would you see on the curve if the wheels of the car were made smaller?
2. How does the gear ratio (ratio of number of teeth of two gears) affect the frequency of curve?







## TRIANGULAR WAVE CAR

## LEARNINGS

Trigonometry

Length of an arc


Trigonometry, LIVE. Sounds vague, doesn't it? Before you say no, pointing out to the many applications of trigonometry in real life; here we are talking about witnessing a triangular wave live in action with the help of a car no less!

## WHAT'S GOING ON?

1. The gear fixed on the axle of the car rotates with the wheels of the car in the same vertical plane.
2. Another gear is joined to this gear in the horizontal plane.
3. A circular (sector of a circle) disc is attached with this gear on the top. Therefore, this disc rotates in the horizontal plane when the car moves in any direction.
4. The stick on the disc moves the module on the top (on which the pen is attached) in a straight line.
5. So the car moves forward and the pen moves in a perpendicular direction to it. So theoretically, the car could make any curve. So why does it make the triangular curve?
6. When the wheel of the car moves $\theta$, the car moves $\mathrm{r} \theta$ ( r is the radius of the wheel) in the forward direction. And the module shifts $\mathrm{R} \theta$ ( R is the radius of the top disc) in the perpendicular direction to the motion of the car.
7. The pen, therefore, draws the graph of $R \theta$ vs $r \theta$ which is a triangular curve $(y=R \theta ; x=r \theta$ $y=R .(x / r)=(R / r)$. $x$, which is the equation of $a$ straight line)








## CYCLOID CAR

## LEARNINGS

Trigonometry

Circle

Rolling of a Circle


This car shows the curve traced by a point on the circumference of a wheel when it rolls on the ground. This is just extremely satisfying to watch and the curves generated are a treat to the eyes!

## WHAT'S GOING ON?

1. A cycloid is the curve traced by a point on the rim of a circular wheel as the wheel rolls along a straight line.
2. In the car, the cycloid curve is generated when the length of the rotating stick is equal to the diameter of the car wheel.
3. You can also change the length of the rotating stick holding the pen and get different kinds of curves (called trochoids).

## EXPLORE

In this case, the gears connecting the axle and the rotating disc have same number of teeth. What would happen to the curve if the number of teeth in both gears is different?









## STRAIGHT LINE DRAWER

## LEARNINGS

Gear System


How do you draw a perfectly straight line by just rotating some gears? Rotate the handle and get ready to be amazed as the pen traces a straight line!
Vertex, Edges and Faces of Polyhedron

## WHAT'S GOING ON?

1. The first gear is fixed at one place and the second and third gears revolve around it.
2. The second gear is half in size as compared to the first gear (the number of teeth in second gear are half of that in first gear).
3. Therefore, if these two gears were fixed at one place, the second gear would have rotated twice as much as the first gear.
4. The second gear is also going around the first gear. As the direction of rotation and revolution is same, the total rotations of the second gear in a single journey around the first gear: $2+1=3$.
5. This means that the second gear rotates three times in one revolution around the first gear.
6. The third gear also goes around the first gear. In this case, the direction of rotation and revolution is opposite. Therefore, total number of rotations: $-2+1=-1$.
7. This means that the third gear rotates one time in one revolution around the first gear, and in opposite direction.
8. The pen is also connected to the third gear and hence rotates once. It also moves in a straight line during this one rotation.

## EXPLORE

Change the distance of the pen from the center of the third gear and see if it still draws a straight line.





## VERTICAL PEN STAND

## LEARNINGS

Magnetism

Simple
Harmonic
Motion

Stability


A magical pen stand - pen stands straight on its tip.
Use the pen and then leave it rotating on its tip on this pen stand!

## WHAT'S GOING ON?

1. The magnets of the pen stand and the pen are placed such that they attract each other.
2. This means that the opposite poles of the magnet (north-south) face each other.
3. How much should be the force of attraction between the magnets to keep the pen vertical? In first look, it looks like the magnetic force should be equal to the weight of the pen.
4. But the magnet is not supporting the weight of the pen. It is just bringing it back to the upright position whenever the pen tilts.
5. When the pen is standing exactly straight on the ground, there is no force required as the force of gravity is balanced by the reaction force from the ground.
6. This is similar to a person standing on the ground. There is no force required to stand straight. But as soon as the person tilts to one side, he needs a force to bring him back to upright position.
7. Similarly, the top magnet brings back the pen in upright position whenever it tries to move away. Therefore the magnetic force required is much smaller than the weight of the pen.


## PULLEY SYSTEM

## LEARNINGS

Tension

Mechanical Advantage


The pulley is a wheel and an axle with a rope going over it. You can put this mechanical advantage to use when you have more than one pulley working together. More the number of pulleys you use, lesser the amount of effort it takes to lift something. The trade-off is that you have to pull the rope proportionately more.

## WHAT'S GOING ON?

Whenever you have to lift a heavy load, say up to the second floor of a building. You could tie a rope to the load, stand on the second floor, and pull it straight up. Or you could put a pulley at the second floor, stand at the first floor, and lift the load by pulling straight down. It would be the exact amount of work in either case, but the action of pulling down feels easier because you're working with the force of gravity. While a single pulley allows you to move a load with half the force required, a system of pulleys increases the mechanical advantage by the number of pulleys and the length of rope that supports the load. As you increase the number of pulleys, you also increase the distance you have to pull the rope. In other words, if you use two pulleys, it takes half the effort to lift something, but you have to pull the rope twice as far. Three pulleys will result in one-third the effort - but the distance you have to pull the rope is tripled! The length of the rope that supports the load in a multiple pulley system basically corresponds to the mechanical advantage of the system.

## EXPLORE

Tie the rope to one of the brooms (broom 1) and wrap the rope around the other broom (broom 2). Have two students stand about a meter apart each holding one broom, and try to keep the brooms separated while the third student pulls on the free end of the rope; it should be a difficult task to pull the broom sticks together. Next, wrap the rope around each of the brooms again. Try again to pull the students/ brooms together; the more times you wrap the rope around the brooms, the easier it is for the third student to pull the others together! This is the power of mechanical advantage!









## GEAR LAMP

## LEARNINGS

Gear
Mechanism
Mechanical
Advantage
Circular to
Linear Motion

Dynamic
Mechanism


Gear Lamp is a mechanical model designed for self-assembly, without any glue. This unique lamp has incorporated various interesting mechanisms. Indulge in the engaging and rewarding experience to build this lamp in a group!

## WHAT'S GOING ON?

1. .The gear system in the lamp is arranged in such a way that it gives mechanical advantage during the motion of the lamp. The handle is attached to a small gear which in turn is connected to larger gears.
2. The whole arm containing the gears also moves along with movements at joints.
3. When the arm rises, the angle of the lamp holder also changes, all from a single source of rotation.

## EXPLORE

Calculate the gear ratio for every gear combination involved in the model. The gear attached to the handle is 4 times smaller than the next connected gear. Therefore, the gear ratio is 4 . Similarly, the next gear assembly has a gear ratio of 4 . So overall the gear lamp has a gear ratio of $4 x$ $4=16$.





















## DUAL OF CUBE

## LEARNINGS

3D shapes

Polyhedrons

Vertex, Edges and Faces of Polyhedron


If you join the center of the faces of a cube, you get the octahedron which is another Platonic solid. This means that the dual of an cube is octahedron.

## WHAT'S GOING ON?

|  | FACES | VERTICES | EDGES |
| :--- | :---: | :---: | :---: |
| SOLID | 6 | 8 | 12 |
|  |  |  |  |
| DUEL | 8 | 6 | 12 |




# DUAL OF OCTAHEDRON 

## LEARNINGS

3D shapes

Polyhedrons
Vertex, Edges and Faces of Polyhedron


If you join the center of the faces of an octahedron, you get the cube which is another Platonic solid. This means that the dual of an octahedron is cube.

## WHAT'S GOING ON?

|  | FACES | VERTICES | EDGES |
| :--- | :---: | :---: | :---: |
| SOLID | 8 | 6 | 12 |
|  |  |  |  |
| DUEL | 6 | 8 | 12 |

## EXPLORE

The solid obtained by joining the center of the faces of a polyhedron is called its dual structure. If you join the center of the faces of a cube, you get a structure called octahedron which is another Platonic solid. This means that the dual of a cube is octahedron.

For any two dual structures, the number of faces and vertices is swapped, and the number of edges is same.



## DUAL OF TETRAHEDRON

## LEARNINGS

3D shapes

Polyhedrons

Vertex, Edges and Faces of Polyhedron


If you join the center of the faces of a tetrahedron, you again get a (smaller) tetrahedron. Therefore, the dual of tetrahedron is a tetrahedron.

WHAT'S GOING ON?

|  | FACES | VERTICES | EDGES |
| :--- | :---: | :---: | :---: |
| SOLID | 8 | 6 | 12 |
|  |  |  |  |
| DUEL | 6 | 8 | 12 |

## EXPLORE

Find out other polyhedrons which are self-duals.



## DUAL OF DODECAHEDRON

## LEARNINGS

3D shapes

Polyhedrons

Vertex, Edges and Faces of Polyhedron


If you join the center of the faces of a dodecahedron, you get the icosahedron which is another Platonic solid. This means that the dual of an dodecahedron is icosahedron.

WHAT'S GOING ON?

|  | FACES | VERTICES | EDGES |
| :--- | :---: | :---: | :---: |
| SOLID | 12 | 20 | 30 |
|  |  |  |  |
| DUEL | 20 | 12 | 30 |




## DUAL OF ICOSAHEDRON

## LEARNINGS

3D shapes

Polyhedrons

Vertex, Edges and Faces of Polyhedron


If you join the center of the faces of an icosahedron, you get the dodecahedron which is another Platonic solid. This means that the dual of an icosahedron is dodecahedron.

WHAT'S GOING ON?

|  | FACES | VERTICES | EDGES |
| :--- | :---: | :---: | :---: |
| SOLID | 20 | 12 | 30 |
|  |  |  |  |
| DUEL | 12 | 20 | 30 |




## ARCHIMEDEAN <br> SOLIDS (13)

## LEARNINGS

3D shapes

Polyhedrons
Dihedral
Angles
Vertex, Edges and Faces of
Polyhedron


In geometry, an Archimedean solid is one of the 13 solids first enumerated by Archimedes. They are polyhedrons composed of regular polygons meeting in identical vertices.

## WHAT TO DO?

Fold the given GSM paper nets on the lines and stick them together using by applying glue to the provided flaps.

## WHAT'S GOING ON?

1. 2000 years ago, Archimedes made a list of polyhedrons composed of regular polygons as faces with vertex symmetry.
2. Surprisingly, he could only find 13 solids which satisfy these two conditions.
3. In all the Archimedean solids, exactly same number and type of polygons are joined at all the vertices, at same dihedral angles (angle between two faces).
4. For example, in a truncated tetrahedron, two hexagons and a triangle meet at all the twelve vertices.

## TRUNCATED TETRAHEDRON

## LEARNINGS

3D shapes

Polyhedrons
Dihedral
Angles
Vertex, Edges and Faces of
Polyhedron


The truncated tetrahedron is created by truncating the vertices off a tetrahedron. Make this Archimedean solid using the net provided and study its properties.

## WHAT'S GOING ON?

1. A tetrahedron has three triangles joining at each vertex.
2. If you cut the vertices at one-third of the original edge length, you get equilateral triangles in place of original vertices of tetrahedron and hexagons in place of original faces. Cutting the edges to one-third is called regular truncation.
3. Therefore, a truncated tetrahedron has four triangular faces and four hexagonal faces.
4. The are total 8 faces, 12 vertices, and 18 edges. Two hexagons and a triangle are meeting at each vertex.
5. There are 4 more faces on the truncated tetrahedron (compared to a regular tetrahedron) due to the four cuts made.
6. The number of vertices and edges are multiplied by three due to the addition of equilateral triangles.



## TRUNCATED CUBE

## LEARNINGS

3D shapes

Polyhedrons

Dihedral
Angles

Vertex, Edges and Faces of
Polyhedron


The truncated cube is an Archimedean solid obtained by cutting the vertices of a cube.

## WHAT'S GOING ON?

1. A cube has three squares joining at each vertex.
2. If you cut the vertices at one-third of the original edge length, you get equilateral triangles in place of original vertices of cube and octagons in place of original faces. Cutting the edges to one-third is called regular truncation.
3. Therefore, a truncated cube has eight triangular faces and six octagonal faces.
4. The are total 14 faces, 24 vertices, and 36 edges. Two octagons and a triangle are meeting at each vertex.
5. There are 8 more faces on the truncated cube (compared to a regular cube) due to the eight cuts made.
6. The number of vertices and edges are multiplied by three due to the addition of equilateral triangles.


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## TRUNCATED OCTAHEDRON

## LEARNINGS

3D shapes

Polyhedrons
Dihedral
Angles
Vertex, Edges and Faces of
Polyhedron

## EXPLORE

Find out why a truncated cube and truncated octahedron have same number of faces, vertices and edges.


The truncated octahedron is an Archimedean solid which has two hexagons and a square meeting at each vertex. It if formed by cutting off each vertex of an octahedron.

## WHAT'S GOING ON?

1. An octahedron has four triangles joining at each vertex.
2. If you cut the vertices at one-third of the original edge length, you get squares in place of original vertices of octahedron and hexagons in place of original faces. Cutting the edges to one-third is called regular truncation.
3. Therefore, a truncated octahedron has six square faces and eight hexagonal faces.
4. There are total 14 faces, 24 vertices, and 36 edges (exactly same as truncated cube). Two hexagons and a square are meeting at each vertex
5. There are 8 more faces on the truncated cube (compared to a regular cube) due to the four cuts made.
6. The number of vertices and edges are multiplied by four due to the addition of squares.


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## TRUNCATED DODECAHEDRON

## LEARNINGS

3D shapes

Polyhedrons
Dihedral
Angles
Vertex, Edges and Faces of
Polyhedron


The truncated dodecahedron is an Archimedean solid. This polyhedron is formed from a dodecahedron by truncating (cutting off) the corners so that the pentagon faces become decagons and the corners become triangles.

## WHAT'S GOING ON?

1. A dodecahedron has three pentagons joining at each vertex.
2. If you cut the vertices at one-third of the original edge length, you get triangles in place of original vertices of dodecahedron and decagons in place of original pentagonal faces. Cutting the edges to one-third is called regular truncation.
3. Therefore, a truncated dodecahedron has 12 decagonal faces and 20 triangular faces.
4. The are total 32 faces, 60 vertices, and 90 edges. Two decagons and a triangle are meeting at each vertex.
5. There are 12 more faces on the truncated cube (compared to a regular dodecahedron) due to the twelve cuts made.
6. The number of vertices and edges are multiplied by three due to the addition of triangles.



## TRUNCATED ICOSAHEDRON

## LEARNINGS

3D shapes

Polyhedrons

Dihedral Angles Vertex, Edges and Faces of Polyhedron


Perhaps the best-known example of a spherical polyhedron, this structure is found in footballs. The ball comprises of 12 pentagons and 20 hexagons.

## WHAT'S GOING ON?

1. An icosahedron has five triangles joining at each vertex.
2. If you cut the vertices at one-third of the original edge length, you get pentagons in place of original vertices of icosahedron and hexagons in place of original faces. Cutting the edges to one-third is called regular truncation.
3. Therefore, a truncated icosahedron has 12 pentagonal faces and 20 hexagonal faces.
4. The are total 32 faces, 60 vertices, and 90 edges (exactly same as truncated icosahedron). Two hexagons and a pentagon are meeting at each vertex.
5. There are 12 more faces on the truncated icosahedron (compared to a regular icosahedron) due to the four cuts made.
6. The number of vertices and edges are multiplied by five due to the addition of squares.



## CUBOCTAHEDRON

A cuboctahedron is created by rectifying a cube (or an octahedron). It has 8 triangular faces and 6 square faces.

## LEARNINGS

3D shapes
Polyhedrons
Dihedral Angles Vertex, Edges and Faces of Polyhedron


## WHAT'S GOING ON?

1. If you cut the vertices of a cube at half of the original edge length, you get equilateral triangles in place of original vertices of cube. You get smaller squares in place of the original squares of the cube. Cutting the edges to half is called rectification.
2. Therefore, a cuboctahedron has eight triangular faces and six square faces.
3. The are total 14 faces, 12 vertices, and 24 edges. Two triangles and two squares meeting at each vertex.
4. There are 8 more faces on the cuboctahedron (compared to a regular cube) due to the eight cuts made.
5. The number of edges in cuboctahedron (24) is triple the number of vertices in cube (8) as each vertex is converted in a triangle, producing three edges for one vertex. The original edges are destroyed in this process.
6. The number of vertices is also tripled due to formation of equilateral triangle at each vertex. But two of these vertices combine together. So the number of vertices in cuboctahedron $=24 / 2=12$.



## RHOMBICUBOCTAHEDRON

## LEARNINGS

3D shapes
Polyhedrons
Dihedral Angles
Vertex, Edges and Faces of
Polyhedron


Johannes Kepler (famous for laws of planetary motion) named this polyhedron a rhombicuboctahedron - short for truncated cuboctahedral rhombus. This solid is made by truncating the cuboctahedron to one-third of its original edge length.

The first printed version of the rhombicuboctahedron was by Leonardo and appeared in Pacioli's work who was collaborator of Da Vinci.

## WHAT'S GOING ON?

1. A cuboctahedron has 12 vertices, with 2 triangles and 2 squares meeting at each vertex.
2. If you cut the vertices of cuboctahedron at half of the original edge length, you get squares in place of original vertices of cuboctahedron. The original squares and triangles of cuboctahedron retain their shape but become smaller.
3. These 12 squares, along with 6 original squares, make a total of 18 squares. The number of triangles remain unchanged at 8 .
4. There are total 26 faces, 24 vertices and 48 edges. Three squares and a triangle are meeting at each vertex.
5. The number of edges in rhombicuboctahedron (48) is four times the number of vertices in cuboctahedron (12) as each vertex is converted in a square, producing four edges for one vertex. The original edges are destroyed in this process.
6. The number of vertices also becomes four times due to formation of squares at each vertex. But two of these vertices combine together. So the number of vertices in rhombicuboctahedron $=48 / 2=24$.



## RHOMBICOSIDODECAHEDRON

LEARNINGS<br>3D shapes<br>Polyhedrons<br>Dihedral Angles<br>Vertex, Edges and Faces of Polyhedron



Johannes Kepler (famous for laws of planetary motion) named this polyhedron a rhombicuboctahedron - short for truncated cuboctahedral rhombus. This solid is made by truncating the cuboctahedron to one-third of its original edge length.
The first printed version of the rhombicuboctahedron was by Leonardo and appeared in Pacioli's work who was collaborator of Da Vinci.

## WHAT'S GOING ON?

1. A cuboctahedron has 12 vertices, with 2 triangles and 2 squares meeting at each vertex.
2. If you cut the vertices of cuboctahedron at half of the original edge length, you get squares in place of original vertices of cuboctahedron. The original squares and triangles of cuboctahedron retain their shape but become smaller.
3. These 12 squares, along with 6 original squares, make a total of 18 squares. The number of triangles remain unchanged at 8 .
4. There are total 26 faces, 24 vertices and 48 edges. Three squares and a triangle are meeting at each vertex.
5. The number of edges in rhombicuboctahedron (48) is four times the number of vertices in cuboctahedron (12) as each vertex is converted in a square, producing four edges for one vertex. The original edges are destroyed in this process.
6. The number of vertices also becomes four times due to formation of squares at each vertex. But two of these vertices combine together. So the number of vertices in rhombicuboctahedron $=48 / 2=24$.

## EXPLORE

1. The 2 nd order rectification (cutting twice at half of the original edge length) is also called cantellation. Therefore, the rhombicuboctahedron can also be called a cantellated cube or octahedron.
2. Rhombicuboctahedron can also be constructed as an expanded cube (the six faces of cube are expanded and the gap is filled with squares and triangles) or an expanded octahedron.



## TRUNCATED CUBOCTAHEDRON

## LEARNINGS

3D shapes

Polyhedrons

Dihedral Angles
Vertex, Edges and Faces of Polyhedron


The truncated cuboctahedron is also named by Kepler as a truncation of a cuboctahedron. It has 12 square faces, 8 hexagonal faces, 6 octagonal faces, 48 vertices and 72 edges.

## WHAT'S GOING ON?

1. A cuboctahedron has 12 vertices, with 2 triangles and 2 squares meeting at each vertex.
2. If you cut the vertices of cuboctahedron at one-third of the original edge length, you get 12 squares in place of original vertices of cuboctahedron. The original squares and triangles of cuboctahedron become octahedron's and hexagons respectively.
3. Therefore, a truncated cuboctahedron has 12 squares, 8 hexagonal and 6 octagonal faces.
4. There are total 26 faces, 48 vertices and 72 edges. A square, hexagon and an octagon are meeting at each vertex.
5. As each of the 12 vertices of cuboctahedron is converted in a square, you get 4 edges for each vertex of cuboctahedron. And the original 24 edges are also intact. Therefore, number of edges in truncated cuboctahedron $(72)=(12 \times 4)+24$.
6. Each of the 12 vertices also produces four vertices after truncation. And the original vertices are destroyed in this process. So the number of vertices in truncated cuboctahedron $=12 \times 4=48$.



## ICOSIDODECAHEDRON

## LEARNINGS

3D shapes
Polyhedrons
Dihedral Angles Vertex, Edges and Faces of Polyhedron


An icosidodecahedron is an Archimedean solid with twenty (icosi) triangular faces and twelve (dodeca) pentagonal faces. An icosidodecahedron has 30 identical vertices, with two triangles and two pentagons meeting at each, and 60 edges

## WHAT'S GOING ON?

1. An icosahedron has five triangles joining at each vertex.
2. If you cut the vertices at half of the original edge length, you get pentagons in place of original vertices of icosahedron. You get smaller triangles in place of the original triangles of icosahedron. Cutting the edges to half is called rectification.
3. Therefore, a cuboctahedron has 12 pentagonal faces and 20 triangular faces.
4. The are total 32 faces, 30 vertices, and 60 edges. Two pentagons and two triangles are meeting at each vertex.
5. There are 12 more faces on the icosidodecahedron (compared to a regular icosahedron) due to the twelve cuts made.
6. The number of edges in icosidodecahedron (60) is five times the number of vertices in icosahedron (12) as each vertex is converted in a triangle, producing three edges for one vertex. The original edges are destroyed in this process.
7. The number of vertices is also tripled due to formation of pentagons at each vertex. But two of these vertices combine together. So the number of vertices in icosidodecahedron $=60 / 2=30$.



## TRUNCATED ICOSIDODECHAHEDRON

## LEARNINGS

3D shapes
Polyhedrons
Dihedral Angles Vertex, Edges and Faces of Polyhedron


The name truncated icosidodecahedron, given originally by Johannes Kepler, is misleading. An actual truncation of a icosidodecahedron has rectangles instead of squares. This nonuniform polyhedron is topologically equivalent to the Archimedean solid

## WHAT'S GOING ON?

1. An icosidodecahedron has 30 vertices, with 2 pentagons and 2 triangles meeting at each vertex.
2. If you cut the vertices of icosidodecahedron at one-third of the original edge length, you get 12 squares in place of original vertices of icosidodecahedron. The original pentagons and triangles of icosidodecahedron become decagons and hexagons respectively.
3. Therefore, a truncated icosidodecahedron has 30 squares, 20 hexagonal and 12 decagonal faces.
4. There are total 62 faces, 120 vertices and 180 edges.
5. As each of the 30 vertices of icosidodecahedron is converted in a square, you get 4 edges for each vertex of icosidodecahedron. And the original 60 edges are also intact. Therefore, number of edges in truncated cuboctahedron $(180)=(30 \times 4)+60$.
6. Each of the 30 vertices also produces four vertices after truncation. And the original vertices are destroyed in this process. So the number of vertices in cuboctahedron $=30 \times 4=120$.



## SNUB CUBE

## LEARNINGS

3D shapes

Polyhedrons

Dihedral Angles Vertex, Edges and Faces of Polyhedron


The final two Archimedean solids are created by moving the faces of an existing Platonic solid outward while giving each face a twist. The snub cube is made from a cube by pulling the square faces outward.

## WHAT'S GOING ON?

1. A cube has three squares joining at each vertex.
2. If you cut the vertices at half of the original edge length, you get equilateral triangles in place of original vertices of cube. You get smaller squares in place of the original squares of the cube. Cutting the edges to half is called rectification.
3. Therefore, a cuboctahedron has eight triangular faces and six square faces.
4. The are total 14 faces, 12 vertices, and 24 edges. Two triangles and two squares meeting at each vertex.
5. There are 8 more faces on the cuboctahedron (compared to a regular cube) due to the eight cuts made.
6. The number of edges in cuboctahedron (24) is triple the number of vertices in cube (8) as each vertex is converted in a triangle, producing three edges for one vertex. The original edges are destroyed in this process.
7. The number of vertices is also tripled due to formation of equilateral triangle at each vertex. But two of these vertices combine together. So the number of vertices in cuboctahedron $=24 / 2=12$.

## EXPLORE

1. The cuboctahedron can also be obtained by cutting the vertices of an octahedron at half of the original edge length.
2. In the Star Trek episode "By Any Other Name", aliens seize the Enterprise by transforming crew members into inanimate cuboctahedron.
3. The "Geo Twister" fidget toy is a flexible cuboctahedron. You can also make this toy by using kebab sticks and cycle valve tubes.
4. The Coriolis space stations in the computer game series Elite are cuboctahedron-shaped.


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## SNUB <br> DODECAHEDRON

## LEARNINGS

3D shapes

Polyhedrons

Dihedral Angles
Vertex, Edges and Faces of Polyhedron


The snub dodecahedron is constructed from a dodecahedron by pulling the pentagonal faces outward.

## WHAT'S GOING ON?

1. A dodecahedron has 12 pentagonal faces.
2. Pull the faces outward so that they no longer touch. Rotate all the pentagons (all clockwise or all counterclockwise) such that the spaces between them can be filled with equilateral triangles.
3. The resulting polyhedron is called snub dodecahedron.
4. There are total 92 faces, 60 vertices and 150 edges. Four triangles and a pentagon are meeting at each vertex.

## EXPLORE

Both of the snub polyhedrons (snub cube and snub dodecahedron) are known as chiral solids. Chiral means that the solids have different forms of handedness which are not mirror-symmetric. Depending on whether you twist the square faces clockwise or counterclockwise, you get two different solids. One form is called left-handed, and the other form is called right-handed.



## NON-TRANSITIVE DICE

## LEARNINGS

Probability


Out of the 3 dice, you can always find a dice which can beat the other one!
In this case, Dice A will beat Dice B, Dice B will beat Dice $C$ but Dice $C$ will beat Dice A. Learn about non-transitive relations using this amazing dice game!

## WHAT TO DO?

1. Take out the pieces from the sheet and make two sets of dice $A, B$, and $C$.
2. For the first part of the game, take only one set (one dice of each kind - A, B and C) and keep the second set aside
3. The first player chooses any one dice from the set and second player would choose from the remaining dice.
4. Both players roll their dice. The player who rolls the higher number wins.
5. Play this game atleast 20 times. Note down the winning dice each time.
6. Try other combinations of dice ( A vs $\mathrm{B}, \mathrm{B}$ vs C and C vs A) and find out which dice is most powerful.

## WHAT'S GOING ON?

1. The dice $A, B$ and $C$ are called non-transitive dice. A set of dice is non-transitive if A is better than $\mathrm{B}, \mathrm{B}$ is better than C but then C is better than A. So the order of the dice is cyclic and there is no single dice which can trump the other two dice. $\mathrm{A}>\mathrm{B}>\mathrm{C}>\mathrm{A}$

2. This is like the game of rock-paper-scissors where scissors can win over paper, rock over scissors and then paper over rock. Just like our dice, no option is the best in this game.
3. If we calculate the probability of A winning over B (and B over C and C over A), it is $7 / 12$. It can be seen by drawing the different outcomes of the dice and seeing which dice would win in each outcome.



CANCE
5/12

C WINS
1/12

C WINS
1/12
4. Now take the second set also in the game. Now you have two copies of each dice A, B, and C. Now each player chooses two dice (both of them should be same. For example, a player can choose both A dice but can't choose one A and one B dice).
5. The sum of two dice would be compared for both the players. In this case, we would see that the cyclic order would get reverses! This means that now $\mathrm{A}<\mathrm{B}<\mathrm{C}<\mathrm{A}$

## EXPLORE

Find out about other non-transitive dice and then make your own set.



## STACK IT UP

## LEARNINGS

Critical
Thinking
Logical Reasoning

Permutation \&Combination

Spatial
Cognition


This puzzle was invented in 1960s and has sold 20 million copies so far under the name "Instant Insanity". The puzzle consists of 4 cubes painted in 4 colors which have to be stacked in a column such that each side of column has all 4 colors.

## WHAT'S GOING ON?

1. You have four cubes with four different colors on each cube. And you have to stack the cubes in a column such that there is a different color on the all the four sides of the column.
2. With 24 positions of each cube ( 6 faces and 4 positions), it makes a total of $3,31,776$ possible arrangements ( $24 \times 24 \times 24 \times 24$ ).
3. Clearly, lot of trial and error is required to get to the solution but there is a way to solve this in eight moves or less by applying graph theory!

## EXPLORE

You can create your own version of the puzzle by coloring the cubes differently. The only constraint is that all cubes should have at least four colors. But out of all these possible versions, only $10 \%$ are solvable. So make your puzzle and try your luck!



## HANDSHAKE PUZZLE

LEARNINGS
Spatial
Thinking
Logical Reasoning

Hand Eye Coordination


From a TV show host to a sweating Professor to a chimpanzee - it took an entire decade before anybody could solve this Japanese ring puzzle. The challenge of the puzzle is to transfer the handshake ring from Student to Teacher. Happy puzzling!

## WHAT'S GOING ON?

1. If you see carefully, the rope goes through the hole to the other side but again comes back to the initial side of the partition.
2. Therefore, it looks impossible at first, but if you keep at it, you will surely succeed!



## RING ROPE PUZZLE



The goal of the puzzle is to remove the rope from structure. The challenge sounds easy but it takes a lot of thinking and playing to solve this! This puzzle is called Baguenaudier puzzle. The word "baguenaudier" means "time-waster" in French. Culin (1965) attributes the puzzle to Chinese general Hung Ming (around 200 A.D.), who gave it to his wife as a present to occupy her while he was away at the wars.

## WHAT'S GOING ON?

1. The goal is to remove the rope from the puzzle.
2. This is called a disentanglement puzzle, a type of mechanical puzzle that involves disentangling one piece of puzzle from another. The reverse problem of reassembling the puzzle can be as hard as - or even harder than-disentanglement.
3. A similar puzzle involving disentangling rings from a rod was originally used by French peasants to lock their suitcases.




## ARROW SLIDING PUZZLE

## LEARNINGS

Spatial
Thinking
Logical
Reasoning
Hand Eye
Coordination


Slide the pieces and make all the arrows point outwards. Interestingly, all the pieces and cells are not the same, which restricts your sliding. Enjoy puzzling!

## WHAT TO DO?

1. At the beginning, all the nine pieces are contained in a $3 \times 3$ grid with arrows pointing inwards.
2. The challenge is to slide the pieces so that all the arrows point outwards.
3. You can only slide the pieces to the empty cells. Notice that there are two types of cells in the puzzle - (i) 9 square-shaped cells in $3 \times 3$ grid and (ii) 2 square cells with rounded corners at the top and bottom of the grid.
4. The puzzle pieces are also of two types. Among the total nine pieces, only 3 have rounded corners which can fit into the top and bottom cells.



## 3 PYRAMIDS TO CUBE

## LEARNINGS

Volume of a Pyramid

Volume of a Cube


The cube can be made using 3 identical pyramids whose volume is $1 / 3$ rd of the cube.

## WHAT'S GOING ON?

1. The three pyramids forming a cube are identical in shape and size. Let's assume that the side of the cube formed by them is 1 unit length.
2. The base of all three pyramids is a square (sides of the square is equal to the side of the cube) and the height of the pyramid is also equal to the side of the cube.
3. The volume of a pyramid is $1 / 3 \times$ (area of base) $\times$ height. Therefore, the volume of one pyramid $=1 / 3 \mathrm{x}$ $(1 \times 1) \times 1=1 / 3$.
4. The pyramids are not right pyramids (pyramids whose apex is above the midpoint of the base).

## EXPLORE

1. The square pyramids have four triangular faces. What is the length of the sides of these triangles?
2. There are two types of triangles - the first type of triangle has sides $(1,1, \sqrt{ } 2)$ and the other one has sides $(1$, $\sqrt{2}, \sqrt{ } 3$ ). Can you identify these two type of triangles in the model?
3. All the triangles forming the pyramids are right triangles!


## 6 PYRAMIDS TO CUBE

## LEARNINGS

Volume of a Pyramid and Cube

Pythagoras Theorem


The six identical pyramids combine to form a cube - which means that volume of one pyramid is $1 / 6$ th of the cube.

## WHAT'S GOING ON?

1. Imagine joining all the vertices of the cube to its center. This will result in dissecting the cube in 6 identical pyramids.
2. The six pyramids forming a cube are identical in shape and size. Let's assume that the side of the cube formed by them is 1 unit length.
3. The base of all six pyramids is a square (sides of the square is equal to the side of the cube) and the height of the pyramid is equal to half of the side of the cube.
4. The volume of a pyramid is $1 / 3 \times$ (area of base) $x$ height. Therefore, the volume of one pyramid $=1 / 3 \mathrm{x}$ $(1 \times 1) \times 1 / 2=1 / 6$.
5. These pyramids are right pyramids (pyramids whose apex is above the midpoint of the base).

## EXPLORE

1. If you invert the six pyramids such that the cube opens inside out, the resulting structure is a rhombic dodecahedron (two triangles come in same place to form rhombuses). This structure will also have a hollow cube inside it. Therefore, the volume of the rhombic dodecahedron is double that of the cube.
2. Find out the length of the triangles making the pyramids (assuming that the side of the cube to be 1 unit).
3. You can also make the cube using six triangular pyramids. Try making these pyramids for yourself.

