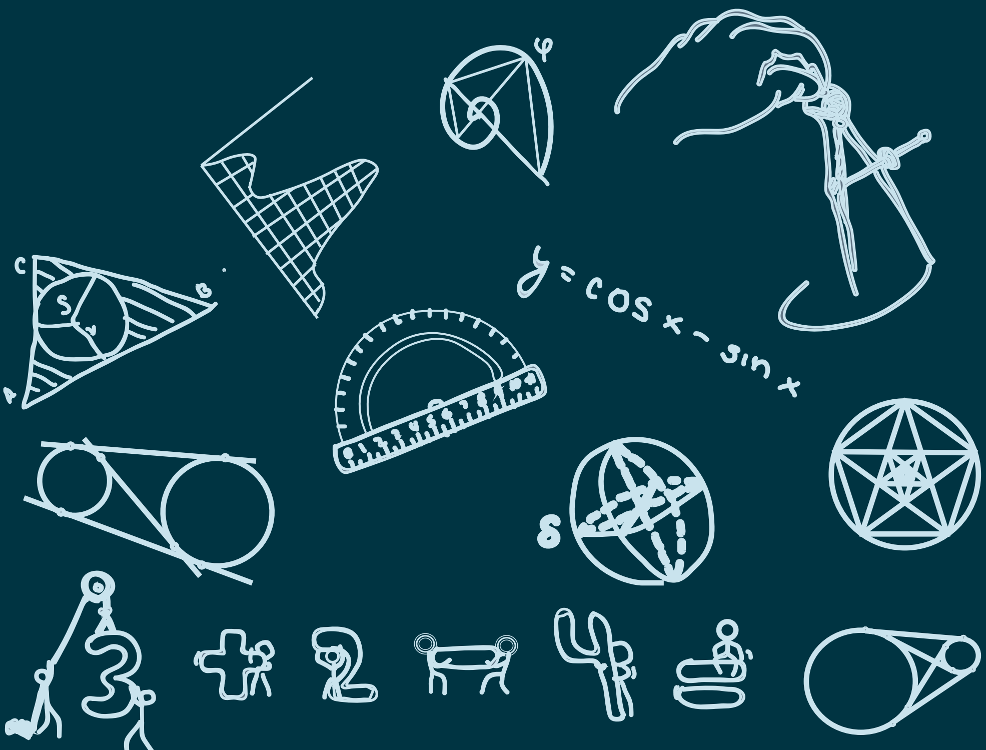
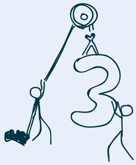
THE WONDERS OF MATHEMATICS

Instructions to your own STEM BOX





SUM OF INTERIOR ANGLES OF A TRIANGLE

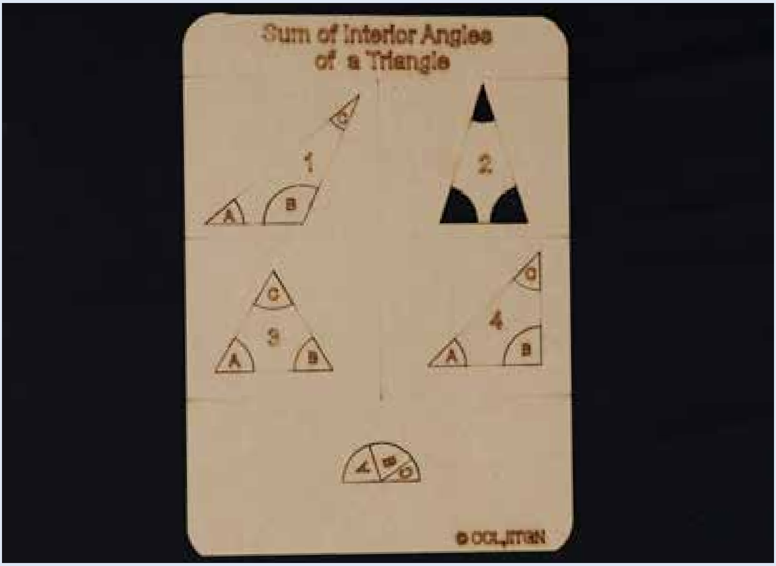
LEARNINGS

Angles of a Triangle

Types of Triangles

Interior Angles

Angle in a Semicircle

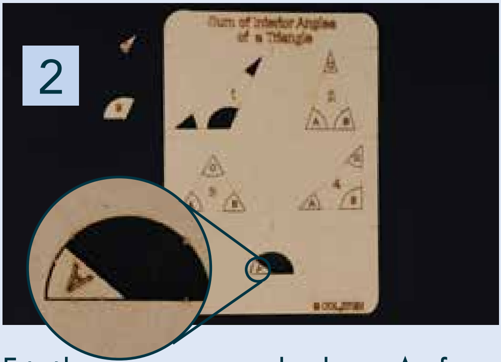


What is the sum of all the angles of a triangle? This activity lets us explore these abstract properties of a triangle and makes them hands-on and exciting.

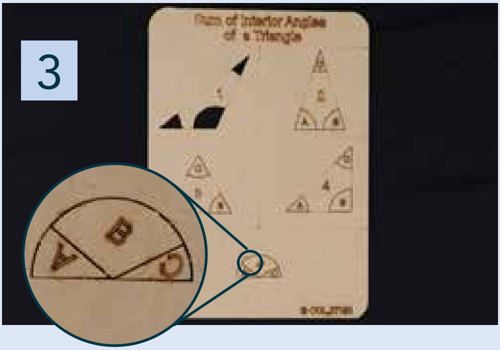
WHAT TO DO?

1. Take out the angles of triangle 1 (marked as A_1, B_1 and C_1) and try to fit it in the semicircle given below.
2. Try with all the four triangles.
3. All the four triangles you see in the sheet look different. But you will see that the angles of all the triangles fit inside the semicircle.

Remove the shown pieces from the sheet and keep the semicircle aside.



Fit the piece marked as A_1 from Triangle 1 in the semicircle, as shown.



Also fit the pieces marked as B_1 and C_1 inside the semicircle.

Repeat step 1 to 3 for all the triangles.

You will see that the parts A, B, C of all the triangles fit inside the semicircle.

NOTES

OBSERVATION

1. The angles cut from the triangles are the arcs of the circle whose radius is equal to the radius of the semicircle given below.
2. There are 360 degrees in a complete circle. The semicircle is half part of a circle. Therefore, the angle in a semicircle is $\frac{360}{2} = 180°$.
3. The three angles of the triangle completely fit inside the semicircle. This shows that the sum of interior angles of the triangles is 180°
4. You can also make your own triangles using cardboard or paper. The angles would always fit in the semicircle, no matter the kind of triangle you make.

EXPLORE

The four triangles given are different types of triangles.Let's see the different ways in which the triangles are classified.

The first way to classify triangles is based on the length of their sides.

- All the three sides of triangle 1 of are unequal in length. Such triangles are called **Scalene triangles**.

- In triangle 2, the length of two sides is same (find out which ones!) and the third side is of different length. Such triangles are called **Isosceles triangles**.

- In triangle 3, the length of all the three sides is same. Such triangles are called **Equilateral triangles**. All the angles of these type of triangles are also equal.

- The fourth triangle is also an Isosceles triangle but a special one. One of the angles (angle B) of this triangle is equal to 90°. Such triangles are called Right triangles. Such triangles have really special properties which are studied in Trigonometry.

NOTES

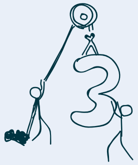
Another way to classify the triangles is based on their angles.

- If all the angles of a triangle are less than 90°, the triangle is called an **Acute triangle**. Triangles 2 and 4 in the sheet are Acute triangles.

- If one of the angles is 90°, the triangle is called a **Right triangle** (Triangle 4). Using the information we have discussed so far, can you prove why two angles in a triangle can't be 90°?

- If one of the angels in a triangle is more than 90°, the triangle is called **Obtuse triangle** (triangle 1). We can never have more than one angle in a triangle more than 90°(convince yourself!).

- If all the angles of a triangle are equal, the three sides are also equal.



SUM OF INTERIOR ANGLES OF A TRIANGLE (VERSION 2)

LEARNINGS

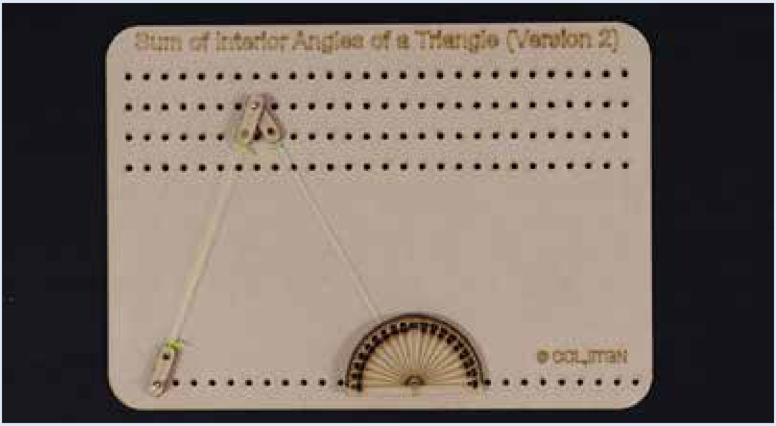
Angles of a Triangle

Types of Triangles

Interior Angles

Exterior Angles

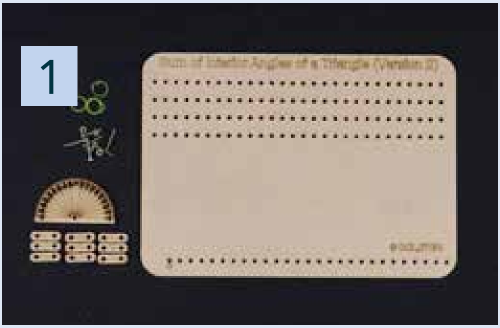
Vertically Opposite Angles



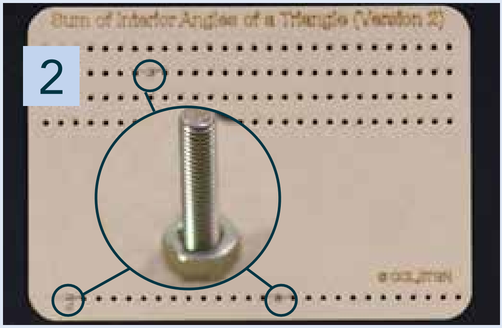
Investigate the relationship between the interior and exterior angles of a triangle in this activity.

WHAT TO DO?

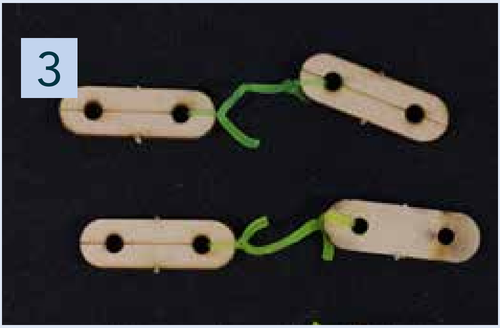
1. Insert bolts in any three holes provided in the sheet and secure them using the nut.
2. Cut a rubber band and join two rounded rectangles with each other by tying rubber band in the holes. Make three such pairs using 6 pointers.
3. Insert the end of these pairs inside the bolts.
4. This would make a triangle with rubber bands as the sides and the bolts as the vertices of the triangle. Instead of rubber bands and pointers, you can also use thread to make the triangle.
5. Using the protractor given with the sheet, measure all the angles of the triangle and add them.
6. Find one of the exterior angles of the triangle (the angle made by the side outside the triangle).
7. Now find the interior angles of the triangle that are opposite to the exterior angle you measured earlier. The sum of these interior angles would always be equal to the exterior angle of the triangle.



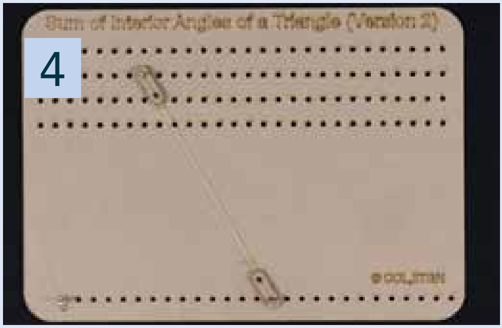
You require the activity sheet, nut-bolts and rubber bands.



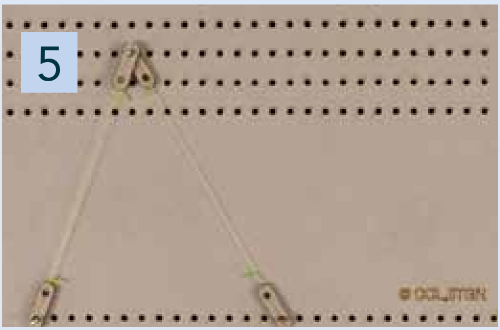
Insert bolts and secure them using the nut as shown.



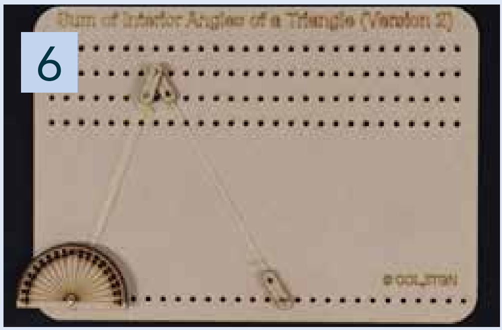
Cut the rubber bands with scissors. Join two pointers with each other by tying rubber bands in the holes.



Use two pointers to make one side of the triangle.



Complete the triangle with the help of another pair.



Using the protractor, measure all the interior and exterior angles of the triangle.

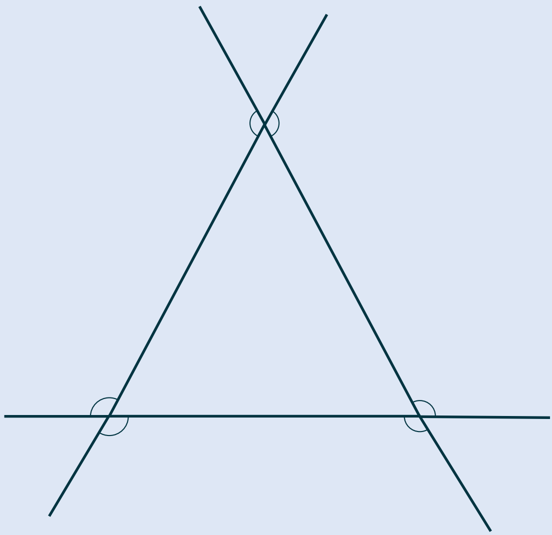
NOTES

OBSERVATION

1. The sum of interior angles always comes out be 180°, no matter the kind of triangle you make.
2. Exterior angle of a triangle is always equal to the sum of two interior opposite angles.

EXPLORE

1. Try and experiment with different types of triangle (acute, obtuse, right angled triangle, equilateral, isosceles, scalene).
2. How many exterior angles are there in a triangle? Try this activity with all the possible combinations of exterior and interior angles.



Six external angles of a triangle. The two exterior angles at any vertex are vertically opposite angles and hence equal.

SUM OF INTERIOR ANGLES OF A QUADRILATERAL

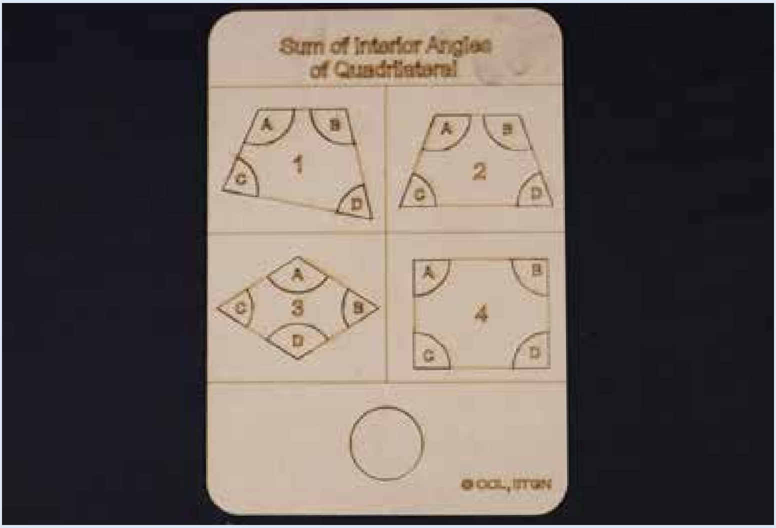
LEARNINGS

Angles of a Quadrilateral

Types of Quadrilaterals

Interior Angles of a Quadrilateral

Angle in a Circle

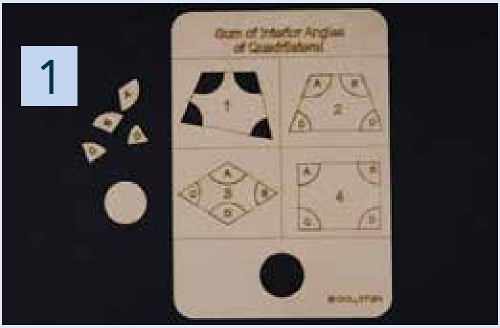


The angles of a quadrilateral can be anything from 0° to 360°. But if you add all the four angles, the answer always comes out same.

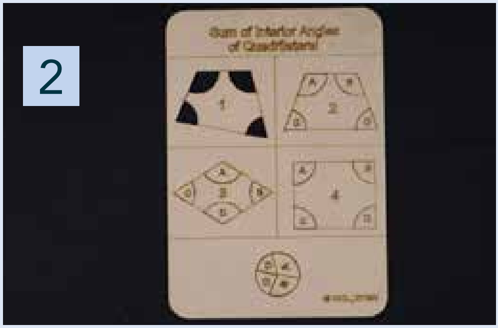
You can check this for four different types of quadrilaterals in this activity and see if their sum is really same!

WHAT TO DO?

1. Take out the angles of quadrilateral 1 (marked as A_1, B_1, C_1 and D_1) and try to fit it in the circle given below.
2. Try with all the four quadrilaterals.
3. All the four quadrilaterals you see in the sheet look different. But you will see that the angles of all the quadrilaterals fit inside the circle.



Take out the angles of quadrilateral 1 (marked as A_1, B_1, C_1 and D_1)



Fit A_1, B_1, C_1 and D_1 inside the circle. You will notice that the parts fit exactly inside the circle.

Take out the angles of quadrilateral 2 as well.

Fit the pieces inside the circle.

Do this for other quadrilaterals also.

You will find that the pieces of all the quadrilaterals fit exactly inside the circle.

NOTES

OBSERVATION

1. The angles cut from the quadrilaterals are the arcs of the circle whose radius is equal to the radius of the circle given below.
2. There are 360° in a complete circle. And the four angles of the quadrilateral completely fit inside the circle.
3. This shows that the sum of interior angles of the quadrilateral is 360°.
4. You can also make your own quadrilaterals using cardboard or paper. The angles would always fit in the circle, no matter the kind of quadrilateral you make.

EXPLORE

The four quadrilaterals given are different types of quadrilaterals.

Let's see the different ways in which the quadrilaterals are classified.

- In quadrilateral 1, all the four sides and angles are different. Such quadrilaterals are called **Scalene quadrilaterals**.

- In quadrilateral 2, there is one pair of opposite sides which are parallel (find out which ones!). Such quadrilaterals are called **Trapeziums**. Sides that are parallel are called bases of the trapezium, and ones that are not parallel are called legs. In fact, quadrilateral 2 is a special kind of trapezium - **Isosceles trapezium** because the two legs are of equal length.

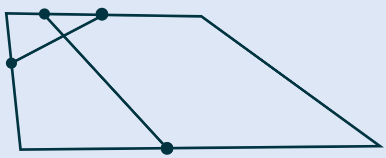
- All the four sides of quadrilateral 3 are equal and it is called a **Rhombus** or an **Equilateral quadrilateral**.

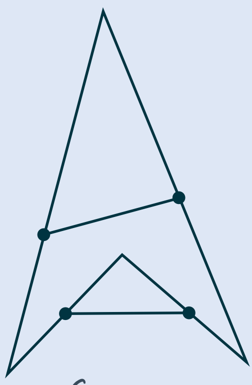
- Quadrilateral 4 is a **Rectangle** in which all the four angles are right angles (90°).

NOTES

The quadrilaterals can also be classified as concave and convex.

- If all the angles are less than 180°, it is called a **convex quadrilateral.** But if any angle is more than 180°, the quadrilateral is called **concave.**
- If you take any two points on the perimeter of a convex quadrilateral, the line joining the two points lie inside the quadrilateral. For a concave quadrilateral, this is not the case. the line joining the points on the circumference can also lie outside the quadrilateral.
- While triangles are always convex, other polygons can be both concave and convex.
- Can you make a quadrilateral in which two angles are more than 180°?
- In case of a triangle, you can always make a circle inside such that it touches all the sides of the triangle. Is this also possible in case of quadrilaterals?


Convex


Concave

SUM OF INTERIOR ANGLES OF A POLYGON

LEARNINGS

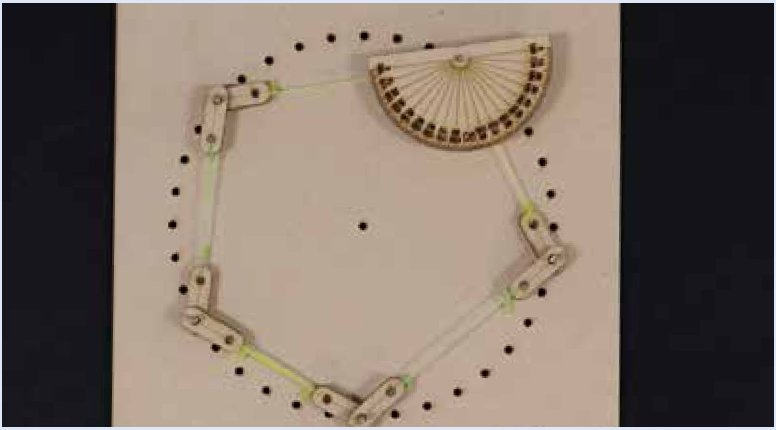
Polygons

Regular Polygon

Irregular Polygon

Interior Angles of a Polygon

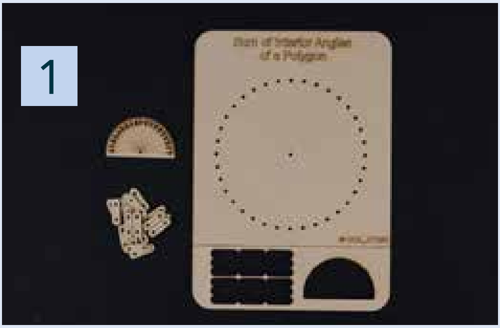
Exterior Angles



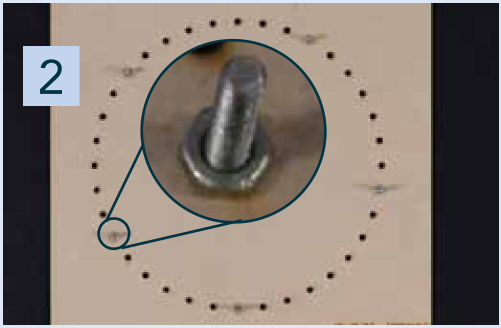
How do you make a regular polygon without using a scale? In this activity, you can build regular (as well as irregular) polygons (triangles, quadrilaterals, pentagons, hexagons etc) and then measure their interior angles.

WHAT TO DO?

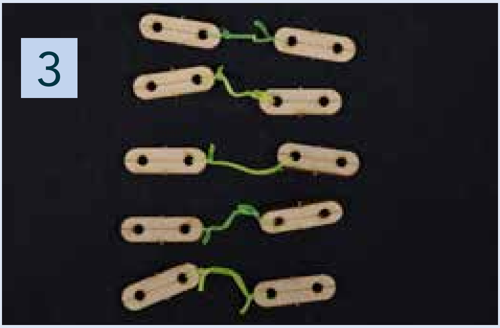
1. There are 36 holes in the sheet forming a circle.
2. For making a triangle, insert bolts in any three holes provided in the sheet and secure them using the nut.
3. Join the bolts using the pointers and rubber bands. Instead of rubber bands and pointers, you can also use thread to join the bolts.
4. For making an equilateral triangle, you have to insert the bolts after every 12 holes.
5. Similarly, for making a square, insert bolts at every 9th hole.
6. You can also make irregular polygons using the sheet.
7. Measure the interior angles of the polygons using the protractors and add them.



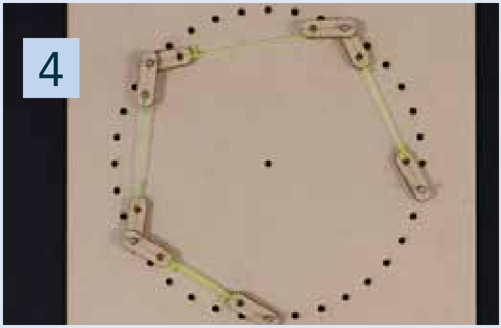
Take out the pieces from the sheet as shown in the picture.



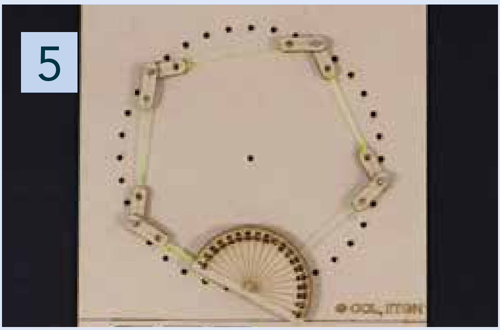
For making a pentagon, insert bolts in any five holes and secure them using the nut.



Join two pointers with each other by tying rubber band in the holes. Make five pairs using 10 pointers.



Make the polygon by joining the bolts with rubber bands.



Measure the angles of the pentagon using the protractor.

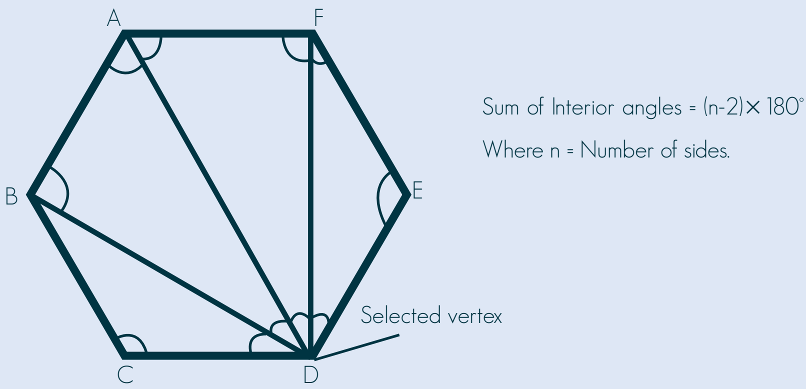
NOTES

OBSERVATION

1. For any polygon, sum of interior angles is (n-2)×180° where n is the number of sides of the polygon.
2. For a triangle, n = 3, because it has 3 sides. Therefore, sum of interior angles = (3 - 2) × 180° = 180°.
3. For a quadrilateral, n=4. Sum of interior angles= (4 - 2) × 180° = 360°.
4. For a pentagon, the sum would be (5 - 2) × 180° = 540°.

WHAT'S GOING ON?

1. For any triangle, the sum of the interior angles is 180°.
2. In any polygon, if you connect any one vertex to all other vertices's, you would get various triangles from the polygon.
3. If you add all the angles of the triangle, it would be (n - 2) × 180°. And if you look closely, all the interior angles of the polygon would also be equal to (n - 2) × 180°.

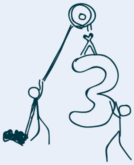


EXPLORE

1. Try making a *regular* pentagon using this sheet.
2. Sum of all the *exterior* angles of any polygon is always 360°. Verify this for different polygons and then try to prove it!

NOTES

3. A regular polygon is a polygon whose sides *as well as* angles are equal. Even if the sides are equal in length, the polygon can be irregular.
4. In this activity, to construct the regular polygons, we only made sure that the sides are equal by inserting bolts at equal intervals. You got regular polygons because the holes were lying on a circle.
5. Can you make an irregular polygon of equal sides having vertices on a circle? For example, a rhombus which is not a square.



PROPERTIES OF A CIRCLE

LEARNINGS

Circle

Chords

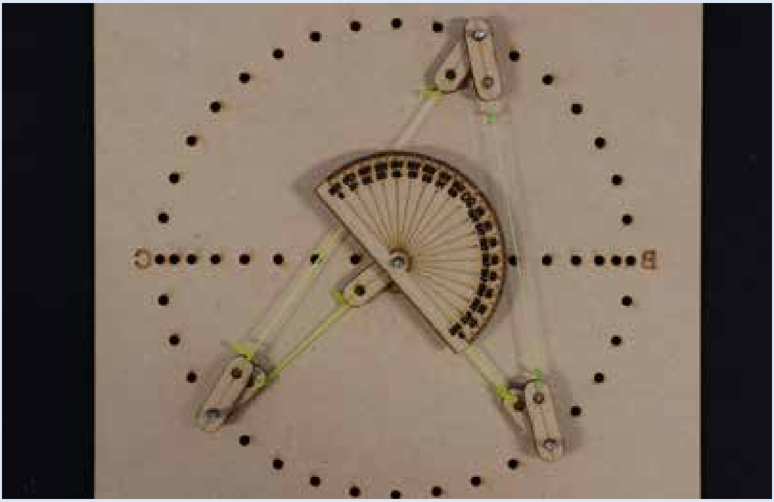
Inscribed Angle

Central Angle

Thales's Theorem

Distance of a Point From a Line

Perpendicular from a Line Segment

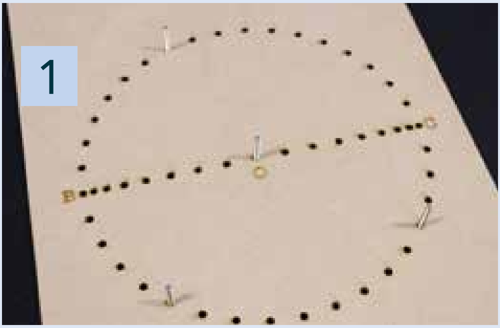


The angle subtended by an arc at the center is always double the angle subtended at any point on the major arc of the circle. The relation holds true for all the points on the circle. You can see this and other properties of a circle by youself in this activity.

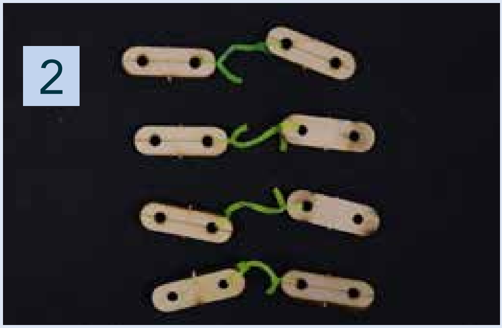
INSCRIBED ANGLE AND CENTRAL ANGLE

1. Insert bolts in any two holes provided at the circumference of the circle and secure them using the nut.
2. Make four pairs of pointers by joining them using rubber bands.
3. Join the points at the circumference to the center of the circle using the rubber bands and pointers. Instead of rubber bands and pointers, you can also use thread to join the points.
4. Take a third point on the major arc and also join this point with the first two points using rubber bands.
5. Now measure the angle at the center of the circle and the angle at the third point using the given protractor.

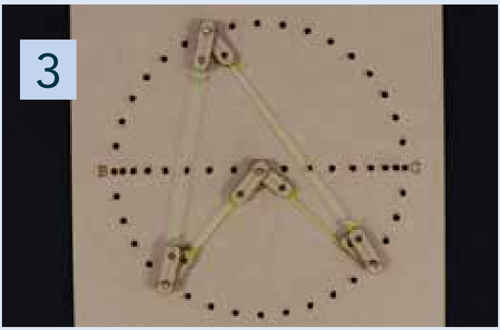
6. Change the third point and again measure all the three angles.
7. You will find that the angle at the center (called the Central angle) is always double that of the angle at the third point (called the Inscribed angle).
8. Inscribed angle would be the same irrespective of the location of the third point.



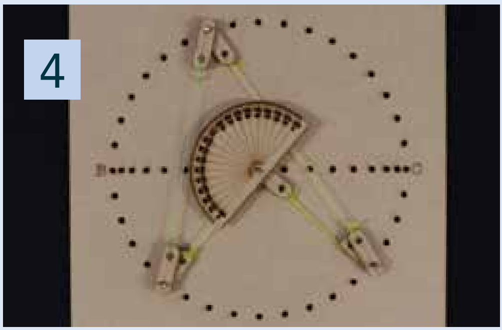
Insert bolts in the four holes as shown in the pictures.



Make four pairs of pointers by joining them using rubber bands.



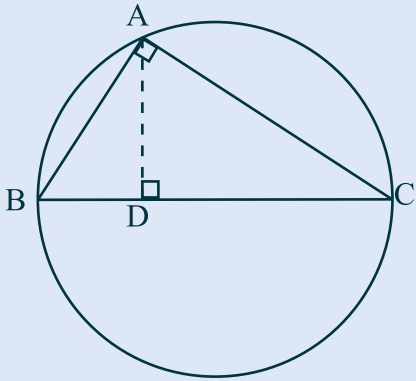
Join the points using rubber bands.



Measure the angles using the protractor.

ANGLE IN A SEMICIRCLE

1. Make a triangle in a semicircle and take diameter BC as the base of the triangle. Take third vertex A on the circumference of the semi circle to make Δ ABC using pointers and rubber bands.
2. Measure $\angle BAC$. It would always be 90° irrespective of where you choose the point A.
3. Now connect point A to the diameter at point D such that AD is perpendicular to BC.
4. Measure the lengths AD,BC and BD.
5. $AD^2 = BD \times DC$



<u>Proof of $AD^2 = BD \times DC$</u>

In ΔABD and ΔACD,

$AB^2 = AD^2 + BD^2$ ----(1)

$AC^2 = AD^2 + DC^2$ ----- (2)

In ΔABC,

$BC^2 = AB^2 + AC^2$

As $BC = BD + DC$,

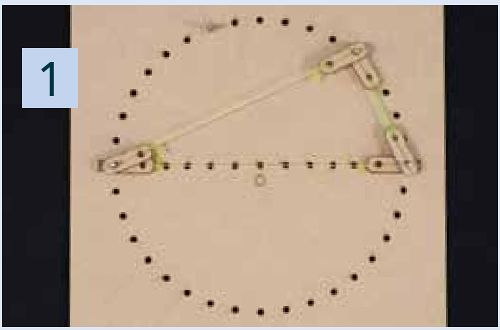
$(BD + DC)^2 = AB^2 + AC^2$

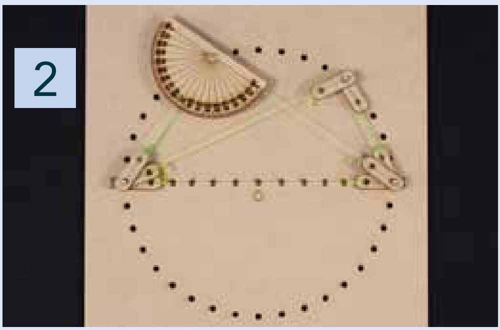
$BD^2 + DC^2 + 2BD \times BC = (AD^2 + BD^2) + (AD^2 + DC^2)$

$2BD \times BC = 2\ AD^2$

Therefore,

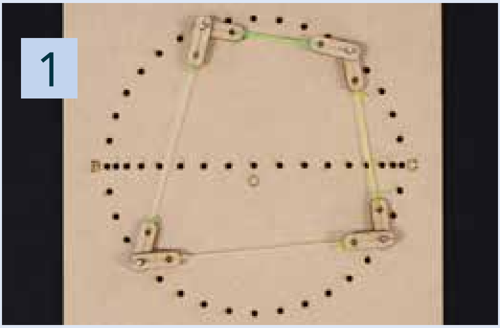
$AD^2 = BD \times DC$

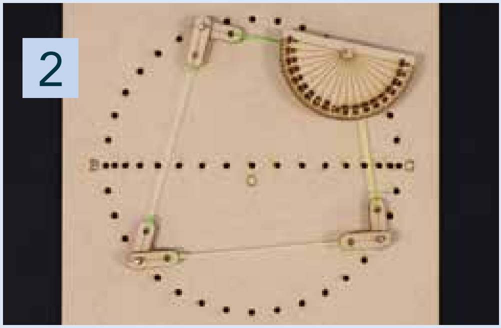


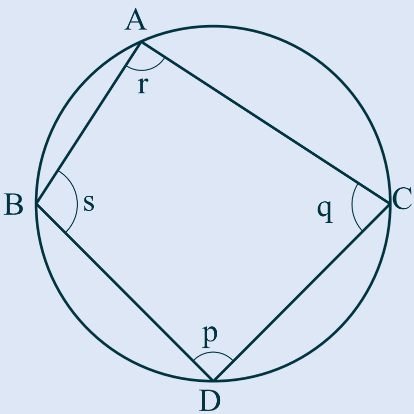


CYCLIC QUADRILATERAL

1. Make a quadrilateral inside the circle using the pointers and rubber bands. This quadrilateral inside a circle is called a Cyclic Quadrilateral.
2. Measure the angles of the quadrilateral using the given protractor.
3. You will find that the sum of opposite angles of the quadrilateral is always 180°. Try this with different types of quadrilateral inside the circle.
4. This is only true for cyclic quadrilaterals, not all the quadrilaterals. Make a random quadrilateral on your notebook and see if the sum of opposite angles is still 180°. You will find that it is not 180°.







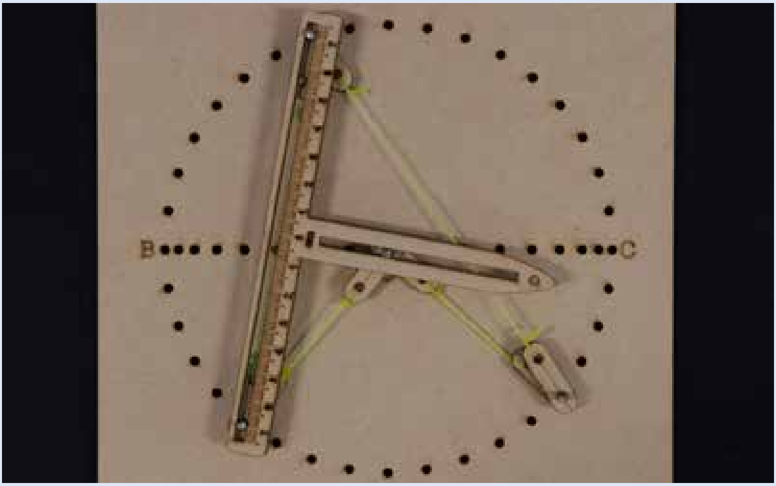
$\angle p + \angle r = 180^\circ$

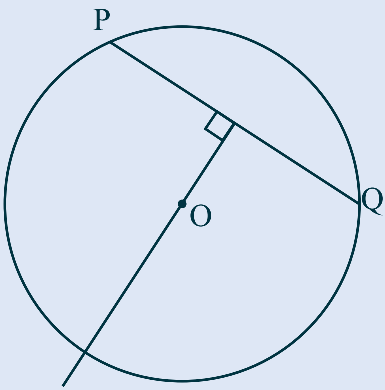
$\angle q + \angle s = 180^\circ$

NOTES

PERPENDICULAR BISECTOR OF A CHORD

1. Make a chord of the circle by joining two points on the circumference using pointers and rubber bands.
2. Place the T along the chord such that markings on both the points of circumference are same. The perpendicular part of the T would pass through the center, regardless of the chord chosen.
3. The T becomes the perpendicular bisector of the chord and the perpendicular bisector of a chord always passes through the center of the circle.





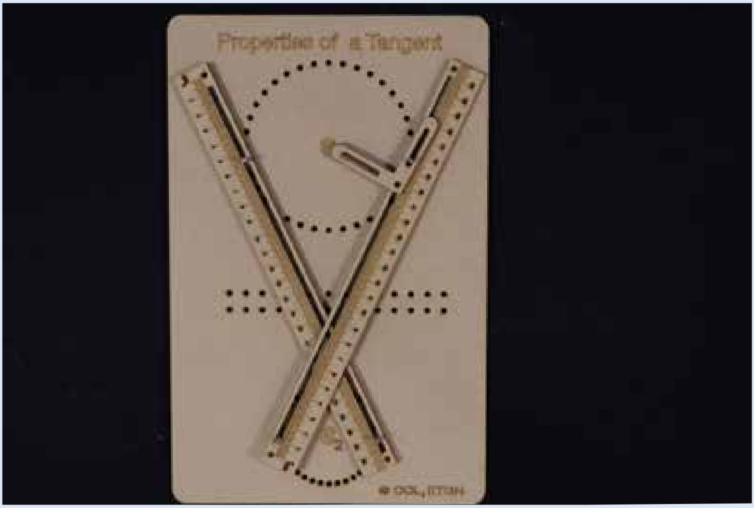
TANGENT FROM AN EXTERNAL POINT

LEARNINGS

Circle

Tangent From a Point to a Circle

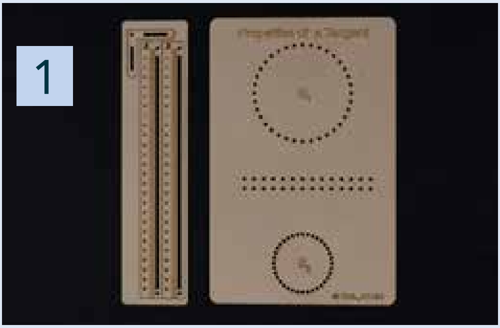
Radius



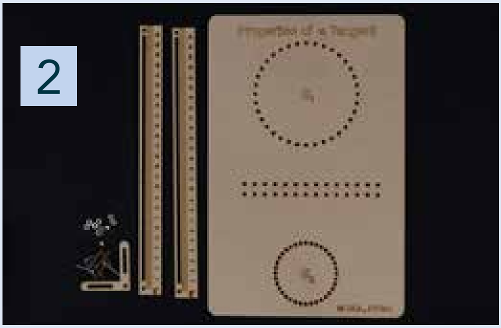
You can draw two tangents to any circle from a point outside the circle. How are these tangents related? Find and compare the lengths of these two tangents in this activity.

WHAT TO DO?

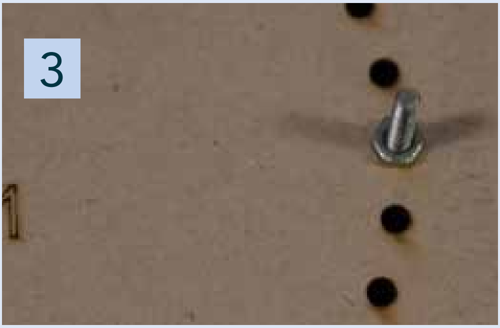
1. Select any outside point from the grid of holes provided in the middle of the sheet.
2. Take the scale given in the sheet and place one end the selected outside point.
3. Now adjust the scale such that it touches the circle at only one point. The scale would be called tangent to the circle in this position.
4. Find the lengths this tangent from the outside point to the point of contact.Similarly, find one more tangent at the other side of the circle andmeasure its lenght. You would find that both the lengths are equal.
5. Place the L piece at the points of intersection of tangents and the circles. The other end of the L would pass through the center of the circle.
6. Now take the scale and place it such that it is tangent to both the circles at the same time.



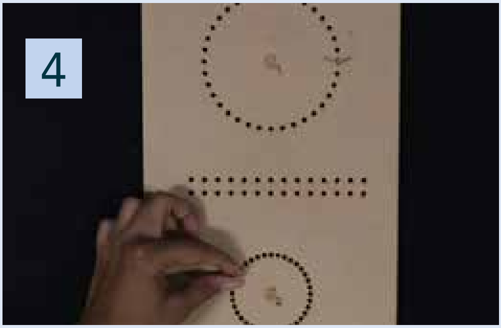
The sheet will contain the following
.



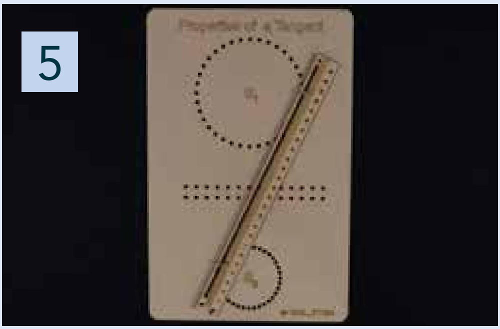
This is what you will need to take from the given material.



Insert 2 screws to make a point on the circle(opposite ends) and fasten it with a screw.

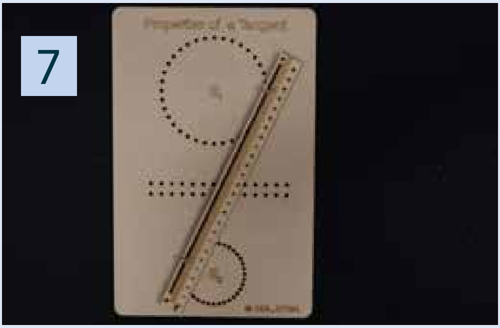


Use two more screws in the smaller circle as well



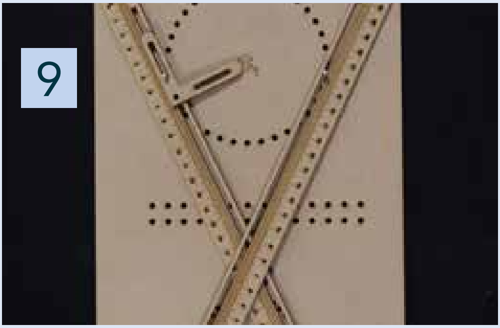
Use the scale given to make the tangent.

Use the screw to attach the scale.



Place the scale such that it is tangent to both the circles at the same time.

This is the second way in which a line can be tangent to both circle.



Add the L shaped piece given along with the sheet. This L will pass through the center of the circle.

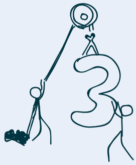
NOTES

WHAT'S GOING ON?

1. A line segment touching the circle at a single point is called tangent to the circle. This tangent line is always perpendicular to the radius of the circle (radius at the point of tangent).
2. You can draw two tangents to a circle from any external point. The lenght of these two tangents is always same.
3. For a set of two circles, you can draw four lines which would be tangents to both the circles at the same time.

EXPLORE

A line segment which cuts the circle at two points is called secant to the circle.
Let's assume that P is the exterior point. From P, the circle has the tangent at points S. If a secant is also drawn such that it intersects the circle at points Q and R.
The length of the tangents and secants are related as follows:
$PS^2 = PQ.PR$
Try this for different tangents and secants in this activity.



CORRESPONDING AND ALTERNATE ANGLES

LEARNINGS

Transversal and Parallel Lines

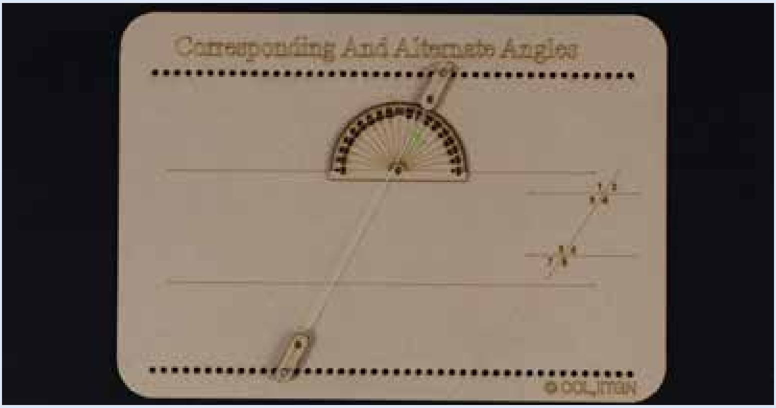
Corresponding Angles

Alternate Angles

Consecutive Interior Angles

Opposite angles

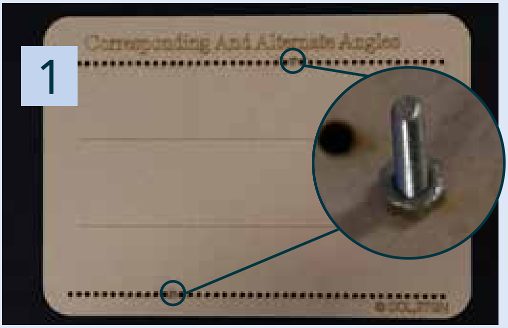
Complementary Angles



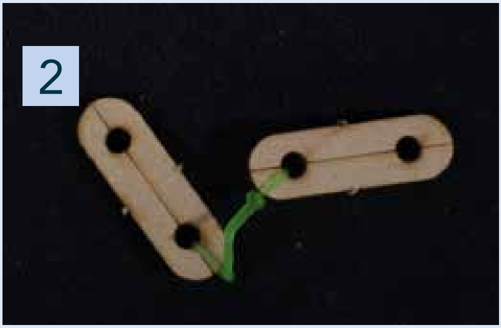
If a line cuts two lines at two different points, it is called a transversal line. If the two lines are parallel, there are some interesting relations between the angles formed between parallel and transversal lines. In this activity, you can measure these angles and see the relation between them for yourself.

WHAT TO DO?

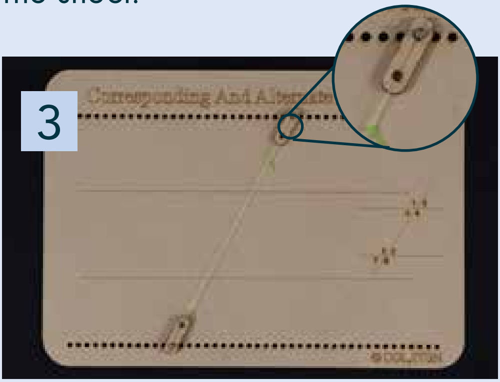
1. Insert bolts in any two holes provided at the top and bottom of the sheet and secure them using the nut.
2. Hold the bolt still by pressing it with your finger (from the back side of the sheet) and rotate the nut in clockwise direction to tighten it.
3. Cut a rubber band and join two pointers (rounded rectangles) with each other by tying rubber band in the holes.
4. Join the two points using the rubber bands and pointers. Instead of rubber bands and pointers, you can also use thread to join the two points.
5. Measure the angles analogous to those marked as 1, 2 , 3 .....8 in the sheet.



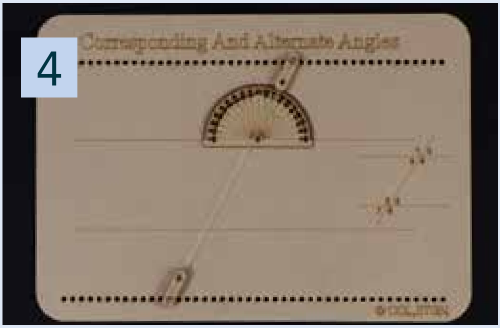
Insert bolts in any two holes provided at the top and bottom of the sheet.



Join two pointers (rounded rectangles) with each other by tying rubber band in the holes.



Fix the pointer to the top and bottom bolt to make the transversal line.



Measure the angles analogous to those marked as 1,2,3.....8 in the sheet.

NOTES

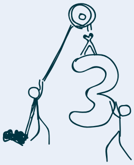
OBSERVATION

1. You would find that $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$. These four pairs of angles are called Corresponding Angles and are always equal.
2. Corresponding Angles lie on the same side of the transversal line and one angle is interior and the other is exterior.
3. $\angle 3 = \angle 6$, $\angle 2 = \angle 7$, $\angle 4 = \angle 5$, $\angle 1 = \angle 8$. These four pairs of angles are called Alternate Angles and are always equal.
4. Alternate Angles lie on opposite sides of the transversal line and both angles are either interior or exterior.
5. $\angle 4 + \angle 6 = 180°$, $\angle 3 + \angle 5 = 180°$. These two pairs of angles are called Consecutive Interior Angles and always add up to 180°.
6. Consecutive Interior Angles lie on the same side of the transversal line.

EXPLORE

1. $\angle 2 + \angle 4 = 180°$ and
   $\angle 1 + \angle 2 = 180°$, as they make a straight line when combined.

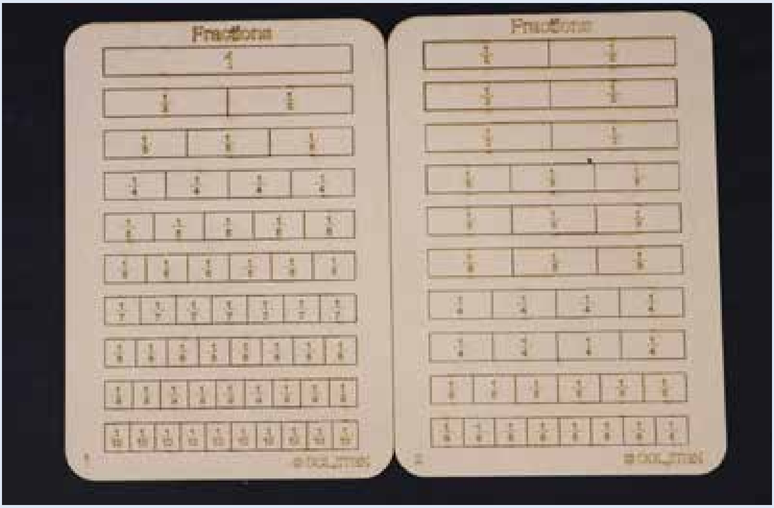
2. $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$ and
   $\angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$, as they make a full circle when combined.



FRACTIONS

LEARNINGS

Introduction to Fractions



In this sheet, you have rectangular pieces which represent different fractions. The pieces can be used to understand and compare fractions in an engaging way.

WHAT TO DO?

1. Take out different pieces and compare the lengths with each other. Which one is the biggest?
2. Neeraj had Rs. 125. He spent 2/5th of the money and saved the rest. How much money did he save?
3. Pankaj had Rs. 500. He spent 1/5th of the money on a storybook and 3/10th on a calendar. How much total money did he spend?
4. Gaurav had Rs 360. He spent 1/3rd of them on Monday and 3/4th of the remainder on Tuesday. How much money did he spend on Tuesday?
5. Use the strips to test your predictions and find out if it's true.

NOTES

OBSERVATION

1. Different fractions are represented by different length of strips. 1> 1/2 > 1/3 > 1/4 > 1/5 > 1/6 > 1/7 > 1/8 > 1/9 > 1/10. The piece marked as 1 is the biggest. All the other pieces are some fraction of this whole.
2. Divide Rs 125 on all the five pieces of 1/5 fraction. Each piece would have Rs 25 on it after this. As Neeraj spent 2/5th of the amount, remove two pieces from this. This means that he spent Rs 50. The remaining amount is corresponding to that present on the three pieces that are left. Therefore, Neeraj saved Rs 75 (do the exercise using the strips and verify this).
3. Divide Rs 500 on all the five pieces of 1/5 fraction. Each piece would have Rs 100 on it after this. As Pankaj spent 1/5th of the amount on a storybook, remove one piece from this. This means that he spent Rs 100 on the storybook. Next, for the calendar, take 3 pieces from the 1/10 fraction. Put these pieces on the 1/5 fraction. This is the amount he spent on the calendar. How much is this amount. You can also see that two 1/10 pieces are covering one piece of 1/5. Therefore, the amount spent on calendar is equal to one complete piece of 1/5 (Rs 100) and one half piece (Rs 50). So the total amount spent on calendar is Rs 150. The total money spent on storybook and calendar combined is Rs 100 + 150 = Rs 250.
4. Divide Rs 360 on all three pieces of 1/3 fraction. Each piece would have Rs 120 on it after this. As Gaurav spent 1/3rd of the amount on Monday, remove one piece from this. This means that he spent Rs 120. The remaining amount is Rs 240. Now you can move ahead in two ways. Either take the remaining amount and divide it equally on all four pieces of 1/4. Each piece would have Rs 60 on it after this. As he spent 3/4th of this on Tuesday, remove three pieces from it. Therefore, the amount spent by Gaurav on Tuesday is Rs 180. (60 + 60 + 60).
5. Or you can keep the remaining money on 1/3 pieces. As one part is out, divide the remaining two parts in four

NOTES

equal parts. Use 1/6 pieces for this. After that, divide Rs 240 on these four parts and proceed as above.

6. Note that the same amount was first represented with 1/4 pieces and then with 1/6 pieces. This was because the whole was different for these two cases. In the first case, the whole was taken as Rs 240, which was the remaining amount. Therefore, 1/4th of this amount was Rs 60. In the second case, the whole was Rs 360 which was the amount he had in the start. Therefore, 1/6th of this amount was also Rs 60.

WHAT'S GOING ON?

1. A fraction is a number that represents a part of a whole. It is written as a numerator displayed above a line (or before a slash), and a denominator, displayed below (or after) that line.
2. For example 2/3 (above and below) or 2/3 (before and after).
3. The numerator represents a number of equal parts, and the denominator indicates how many of those parts make up a unit or a whole.
4. For example, in the fraction 3/4, the numerator, 3, tells us that the fraction represents 3 equal parts, and the denominator, 4, tells us that 4 parts make up a whole.
5. By this definition, we can see that 2/3 would be bigger than 2/4 because in both the fractions, we have 2 equal parts but in 2/3 the whole is divided into 3 equal parts while in 2/4, it is divided into 4 equal parts.

EXPLORE

1. In a village, 2/3rd of males are married to 2/5th of the females. And one male marries one female and vice versa. What fraction of the total population is unmarried in this village?
2. In another village, 2/3rd of males are married to 3/4th of the females. And one male marries one female and vice versa. What fraction of the total population is unmarried in this village?

AREA OF A TRIANGLE (ACUTE)

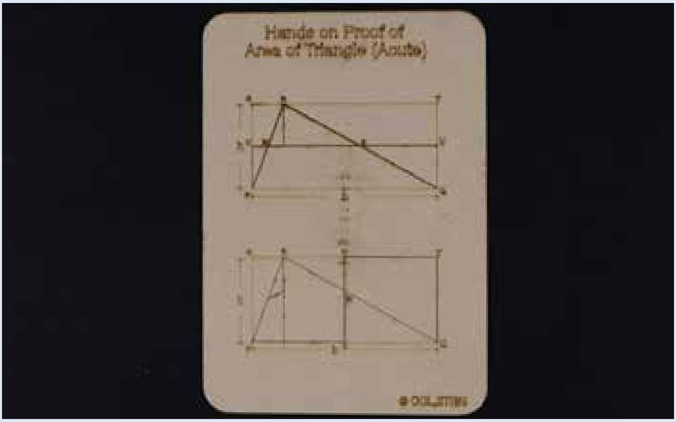
LEARNINGS

Mensuration

Area of a Triangle

Area of a Rectangle

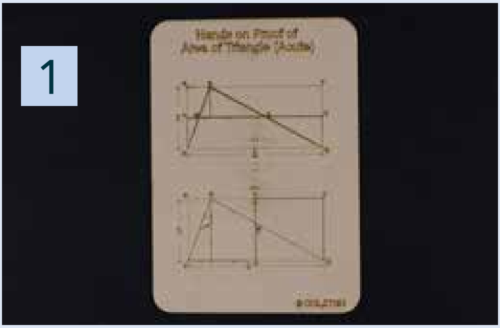
Spatial Thinking



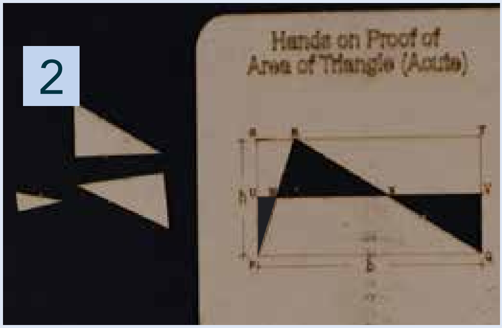
The area of a triangle with base b and height h is (1/2 × b × h). And the area of a rectangle with sides b and h is b × h. This means that the area of a triangle is half of the area of a rectangle, having same base and height.
In this activity, you can check the relationship between the area of a triangle and rectangle with some puzzle pieces. In this sheet, the triangle is an *acute* triangle.

WHAT TO DO?

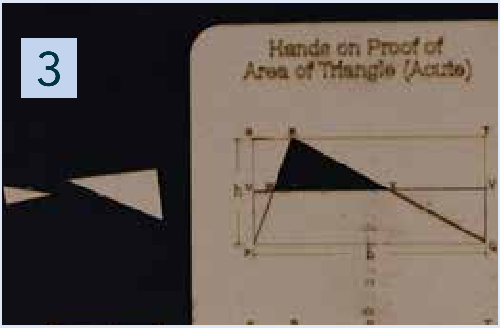
1. The triangle PQR also has base b and height h.
2. The rectangle PQTS also has length and breadth equal to b and h respectively. Thus the area of the rectangle is equal to b × h.
3. Rectangle 1 is divided in half by the line UV along the length. Thus, the area of the rectangle PQVU is equal to 1/2 × (bh).
4. Take out the pieces of triangle PQR and try to fit all of them inside the rectangle PQVU.
5. You will find that the triangle would fit exactly inside the rectangle PQVU and hence has same area as the rectangle.
6. Do this exercise for Rectangle 2. This time, the rectangle is divided in half by the line UV along the breadth. Fit all the pieces of the triangle PQR in the area UTQV.



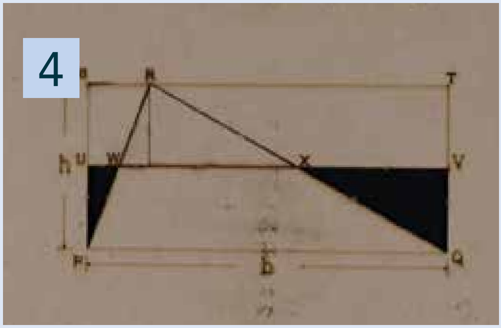
Δ PQR and rectangle PQTS both have length and breadth equal to b and h respectively.



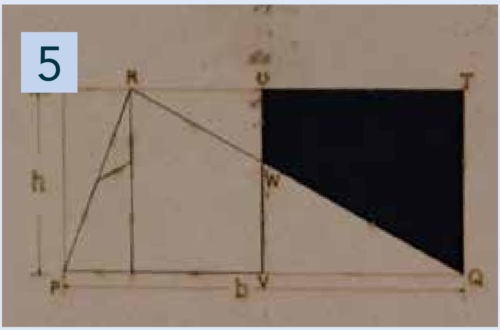
Take out all the pieces of ΔPQR and try to fit all of them inside the rectangle PQVU.



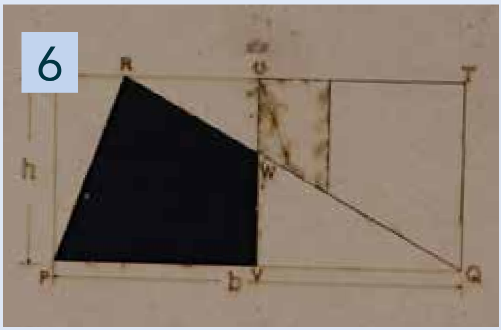
You will find that the triangle fits exactly inside the rectangle PQVU and hence has same area as the rectangle.



Rectangle 1 is divided in half by the line UV along the length. Thus, the area of the rectangle PQVU is half of rectangle PQTS = 1/2x(bh).



Do this exercise for Rectangle 2. This time, the rectangle is divided in half by the line UV along the breadth.



The pieces of ΔPQR fit inside the rectangle QTUV.

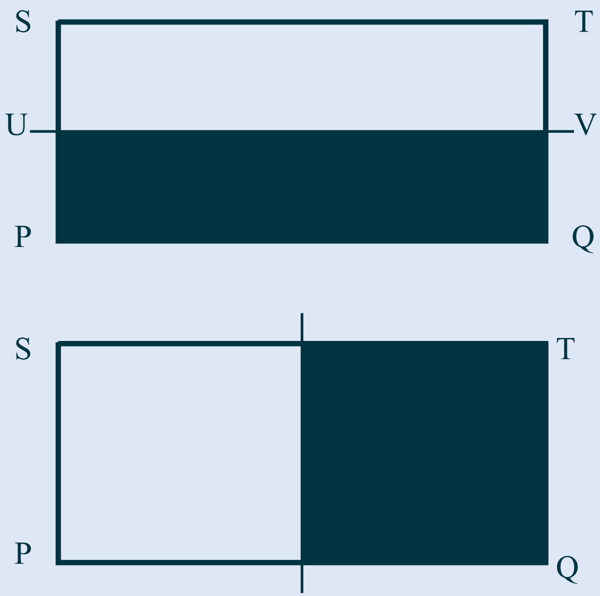
NOTES

OBSERVATION

1. You can see that the pieces of the triangle PQR fit exactly into half of the rectangle PQTS in both the cases.
2. This means that the area of the triangle is half of the rectangle PQTS = $1/2 \times (bh)$.
3. The rectangle is divided in half in two different ways, along the length (Rectangle 1) and along the breadth (Rectangle 2)

EXPLORE

1. Make your own triangles acute angled, right angled and obtuse angled triangles using cardboard or paper and try to fit them inside the rectangles
2. Make sure that the base and height of the triangle and the rectangle is same, otherwise they won't fit.



Two ways of dividing a rectangle in half.

AREA OF A TRIANGLE (OBTUSE)

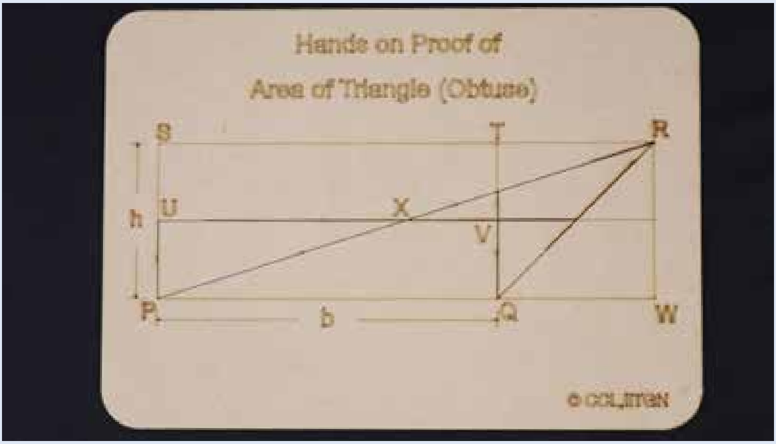
LEARNINGS

Mensuration

Area of a Triangle

Area of a Rectangle

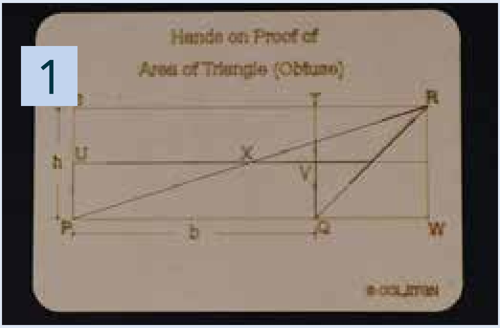
Spatial Thinking



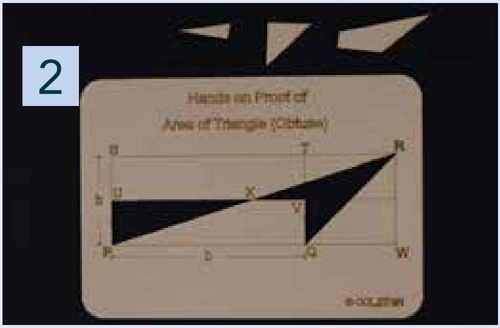
The area of a triangle with base b and height h is 1/2 × (bh). And the area of a rectangle with sides b and h is b × h. This means that the area of a triangle is half of the area of a rectangle. In this activity, you can check the relationship between the area of a triangle and rectangle with some puzzle pieces. In this sheet, the triangle is an Obtuse triangle.

WHAT TO DO?

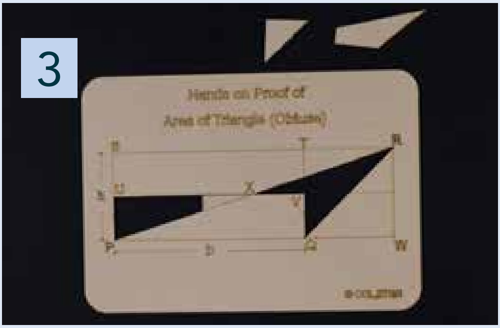
1. The triangle PQR also has base b and height h.
2. The rectangle PQTS has length and breadth equal to b and h respectively. Therefore the area of the rectangle is equal to b × h. Note that we have taken rectangle PQTS and not the whole rectangle PWRS. Because the length and breadth of the rectangle should be equal to the base and height of the triangle.
3. The rectangle is divided in half by the line UV. Therefore, the area of the rectangle PQVU is equal to 1/2 × (bh).
4. Take out the pieces of triangle PQR and try to fit all of them inside the rectangle PQVU.
5. You will find that the triangle would fit exactly inside the rectangle PQVU and hence has same area as the rectangle.



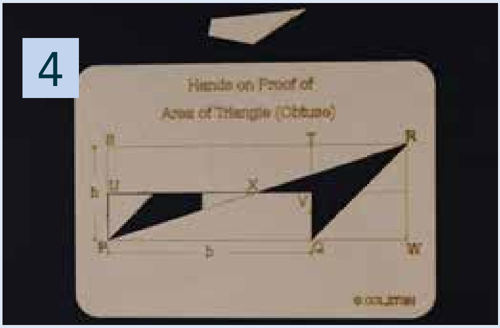
The triangle PQR & rectangle PQTS have base b and height h



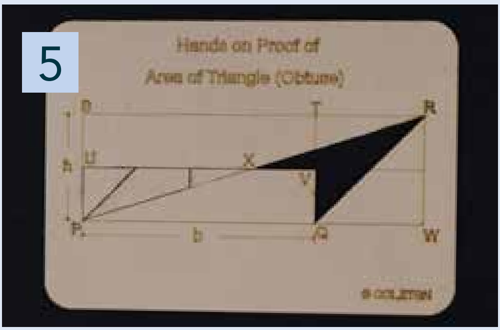
Take out the pieces of ΔPQR and try to fit all of them inside the rectangle PQVU.



The image shows the placement of pieces into the rectangle PQVU.



Rectangle PQTS is divided in half by the line UV. Thus, the area of the rectangle PQVU is equal to ½(bh).



All the triangle pieces would fit exactly inside the rectangle PQVU

NOTES

OBSERVATION

1. You can see that the pieces of the triangle PQR fit exactly into half of the rectangle PQTS
2. This means that the area of the triangle is half of the rectangle PQTS - 1/2 × (bh).

EXPLORE

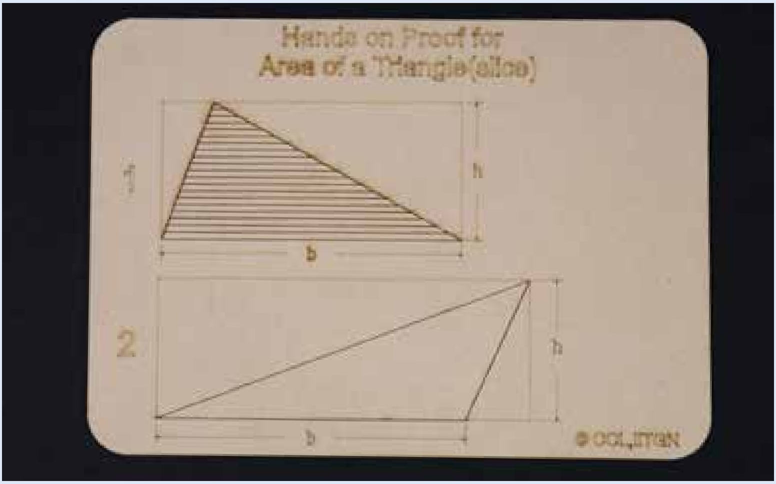
1. Make your own triangles acute angled, right angled and obtuse angled triangles using cardboard or paper and try to fit them inside the rectangles
2. Make sure that the base and height of the triangle and the rectangle is same, otherwise they won't fit.

AREA OF A TRIANGLE (SLICE)

LEARNINGS

Mensuration

Area of a Triangle

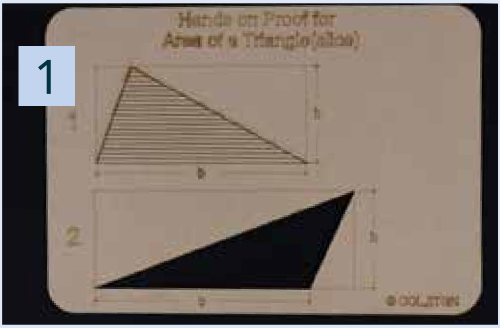


The area of a triangle is $\frac{1}{2}$ (base x height). This means that the area of all the triangles having same base and height should be same, no matter the shape of the triangle.

In this activity, you can verify that the area of two triangles having same base and height is same, even though they look very different.

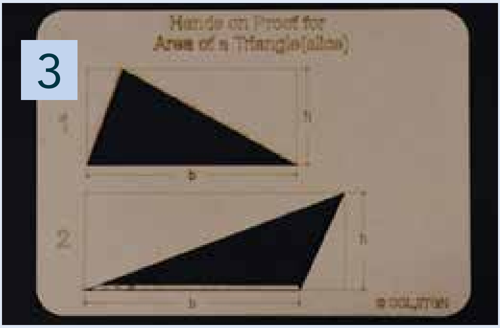
WHAT TO DO?

1. One of the triangles (triangle 1) has been sliced into thin strips. Take the strips of the triangle out of the sheet.
2. Remove the triangle from the bottom of the sheet (triangle 2).
3. Try to fit the slices of triangle 1 into triangle 2.
4. Even though the shape of the triangles is different, the strips of one triangle would fit exactly inside the other triangle

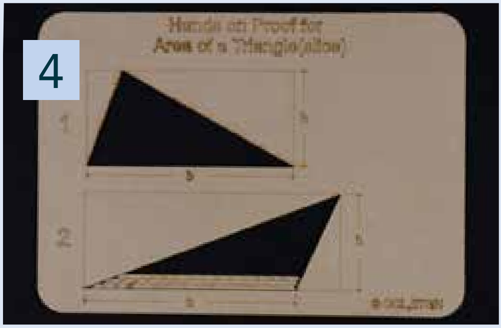


Remove the triangle from the bottom of the sheet.

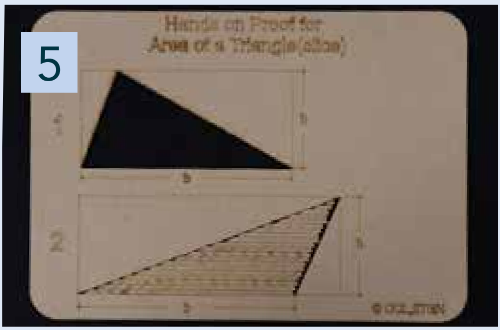
Take the strips of the triangle out of the sheet.



Try to fit the slices of triangle 1 into triangle 2.



You may have to turn over the strips in order to put them in the triangle below.



Even though the shape of the triangles is different, the strips of one triangle fit exactly inside the other triangle.

NOTES

WHAT'S GOING ON?

1. As the base and height of both the triangles are same, their areas should also be same (as area = $1/2 \times b \times h$).
2. The triangle at the top (triangle 1) is divided into strips. All the strips taken together represent the area of the triangle.
3. As the area of the triangles is equal, the strips of one fit exactly inside the other.

EXPLORE

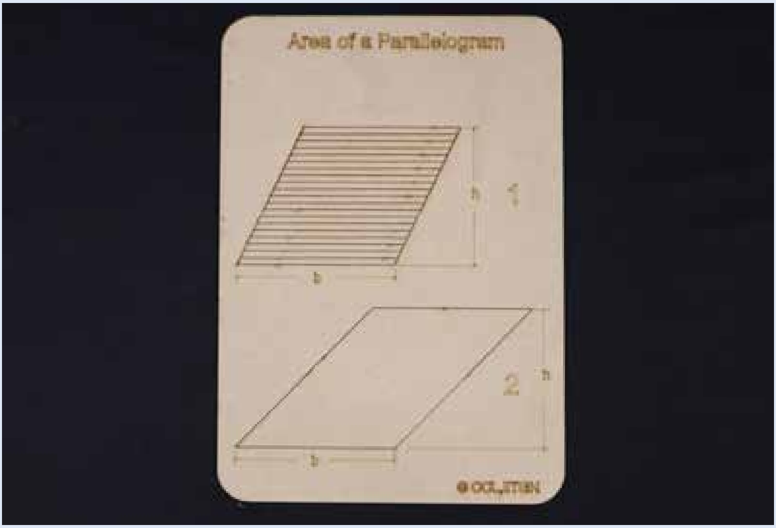
1. You can also put the strips on the floor and push them with a scale to make a different triangle. The base and height of any triangle you make like this would be same.
2. One more way to verify that the area of triangles is same is to fill the space with seeds (or daal) and then see if the same amount of seeds can cover the other triangle. Make sure that the seeds are not stacked on top of each other while doing this.
3. You can also use kebab sticks for this activity.

AREA OF A PARALLELOGRAM

LEARNINGS

Mensuration

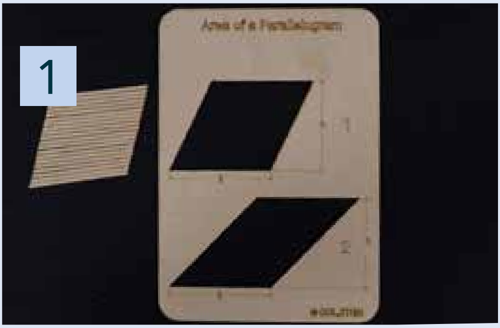
Area of a Parallelogram



All the parallelograms of same base and same height have equal areas (equal to b × h) .The two parallelograms in this activity don't look the same. However, they have equal base, height and hence equal area. The strips of the first parallelogram fit exactly in the other one.

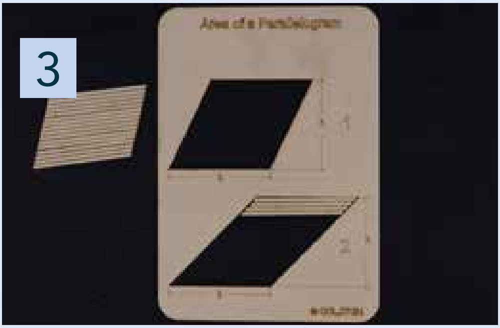
WHAT TO DO?

1. One of the parallelograms (parallelogram 1) has been sliced into thin strips. Take the strips of the parallelogram out of the sheet.
2. Remove the parallelogram from the bottom of the sheet (parallelogram 2).
3. Try to fit the slices of parallelogram 1 in parallelogram 2.
4. Even though the shape of the parallelograms is different, the strips of one parallelogram would fit exactly inside the other parallelogram.



Remove the strips from the top and the parallelogram from the bottom of the sheet (parallelogram 2).

Separate all the strips of parallelogram 1.



Fit the slices of parallelogram 1 in parallelogram 2. You may have to flip the strips to fit.

Even though the shape of the parallelograms is different, the strips of one parallelogram fit exactly inside the other parallelogram.

NOTES

WHAT'S GOING ON?

1. As the base and height of both the parallelograms are same, their areas should also be same (as area = b × h).
2. The parallelogram at the top (parallelogram 1) is divided into strips. All the strips taken together represent the area of the parallelogram.
3. As the area of parallelograms is equal, the strips of one fit exactly inside the other.

EXPLORE

1. You can also put the strips on the floor and push them with a scale to make a different parallelogram. The base and height of any parallelogram you make like this would be same.
2. One more way to verify that the area of parallelograms is same is to fill the space with seeds (or daal) and then see if the same amount of seeds can cover the other parallelogram. Make sure that the seeds are not stacked on top of each other while doing this.
3. You can also use kebab sticks for this activity.

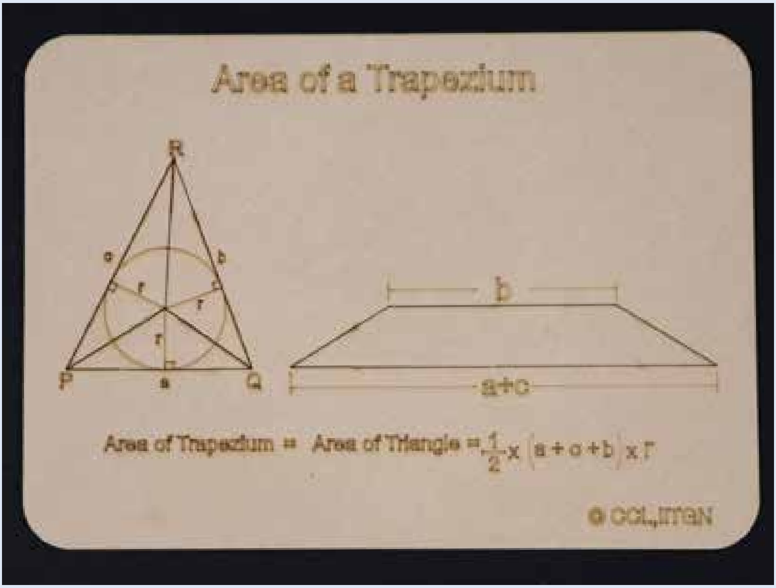
AREA OF A TRAPEZIUM

LEARNINGS

Mensuration

Area of a Trapezium

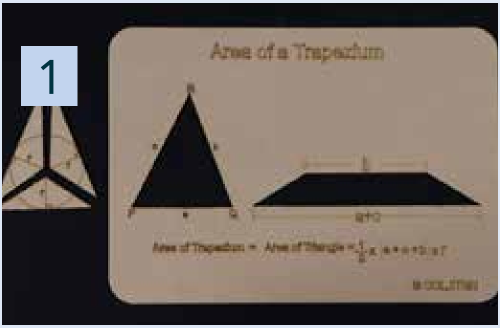
Area of a Triangle



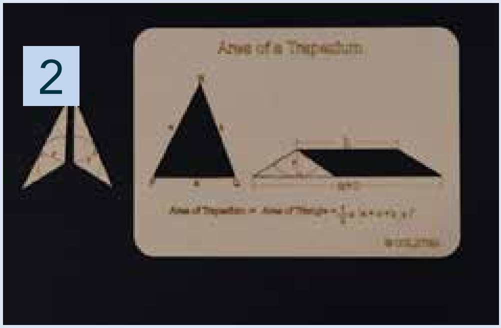
Area of a trapezium is 1/2 × (sum of parallel sides) × height. But how do you arrive at it? In this activity, you can derive this formula, starting from the area of a triangle.

WHAT TO DO?

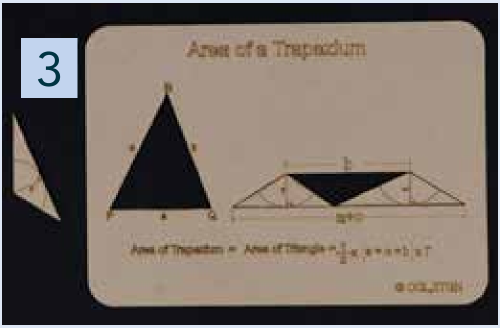
1. Remove the trapezium from the right side of the sheet.
2. Take out the pieces from the triangle, at the left side of the sheet.
3. Fit these pieces of the triangle inside the trapezium (you may need to invert some pieces).
4. You will find that the pieces of the triangle fit exactly inside the trapezium.



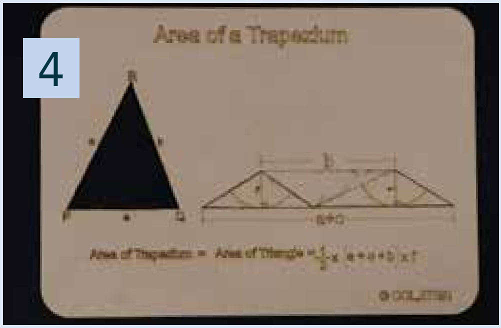
Remove the trapezium and triangle from the sheet.



Fit the pieces of the triangle inside the trapezium.



You may need to invert some pieces.



You will find that the pieces of the triangle fit exactly inside the trapezium.

NOTES

WHAT'S GOING ON?

1. The area of a triangle is equal to 1/2 × base ×height. The bases of the small triangles that make triangle PQR are a, b, and c respectively and height is equal to r.
2. Therefore, the areas of the small triangles would be (1/2 × a × r), (1/2 × b × r) and (1/2 × c × r).
3. The area of triangle PQR is therefore (1/2 × a × r) + (1/2 × b × r) + (1/2 × c × r) = 1/2 (a+b+c) × r
4. As the pieces of triangle PQR completely fit inside the trapezium, it means that their areas are equal.
5. Area of the trapezium = Area of triangle = 1/2 × (a+b+c) × r

   = 1/2 × (a+c + b) × r

   = 1/2 × (sum of parallel side of trapezium)× height

EXPLORE

Can you calculate the value of r in terms of a, b and c ? This is called Heron's formula.

$r = \sqrt{[(s-a)(s-b)(s-c)/s]}$, where $s = (a+b+c)/2$

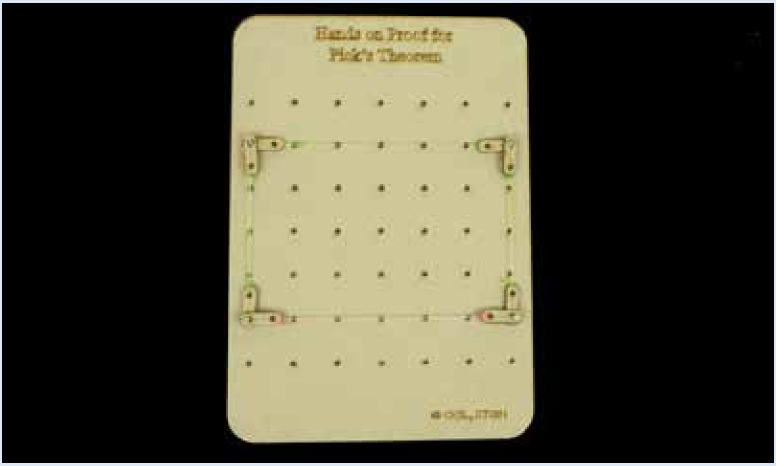
AREA OF A POLYGON
(PICK'S THEOREM)

LEARNINGS

Mensuration

Area of a Triangle

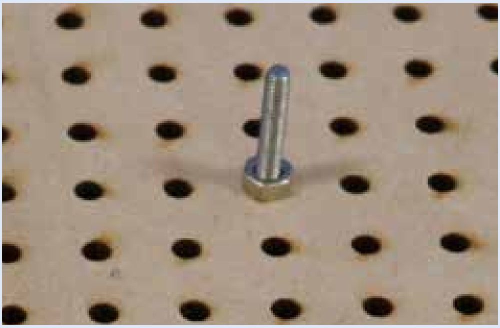
Area of a Polygon



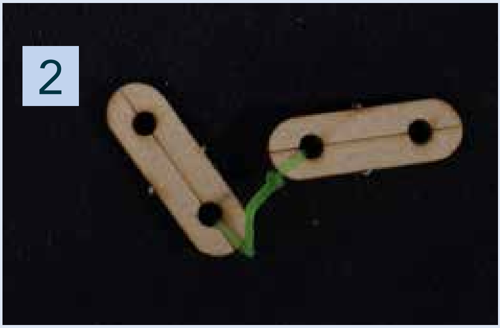
Given a simple polygon constructed on a grid of equal-distanced points such that all the polygon's vertices are grid points, this activity provides a simple formula for calculating the area of the polygon based on the number of points it encloses/touches. Make any polygon with this sheet and find out the area using the wonderful Pick's Theorem!

WHAT TO DO?

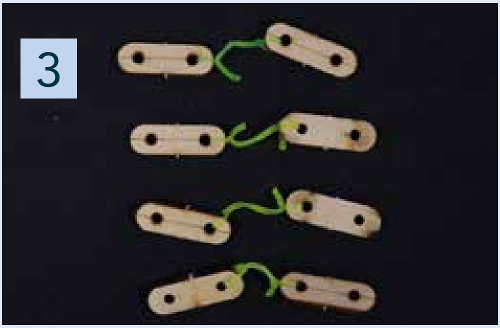
1. Insert bolts in the holes provided in the sheet and secure them using the nut. Hold the bolt still by pressing it with your finger (from the back side of the sheet) and rotate the nut in clockwise direction to tighten it.
2. Cut a rubber band and join two pointers (rounded rectangles) with each other by tying rubber band in the holes. Make a polygon by inserting such pairs of pointers in the bolts. Instead of rubber bands and pointers, you can also use thread to join the bolts.
3. If i is the number of holes inside the polygon and b is the number of holes touched by the rubber bands (points on the polygon's perimeter), the area of the polygon is
   $A= i + b/2 - 1$



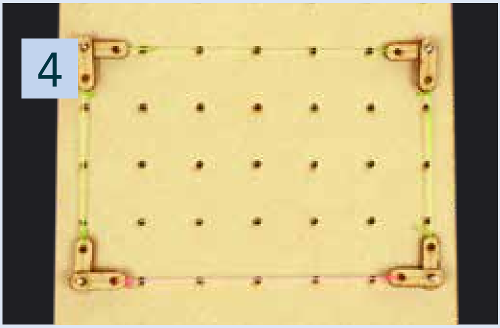
Insert bolts in the holes provided in the sheet and secure them using the nut.



Join two pointers (rounded rectangles) with each other by tying rubber band in the holes.



Make multiple such pairs of pointers.

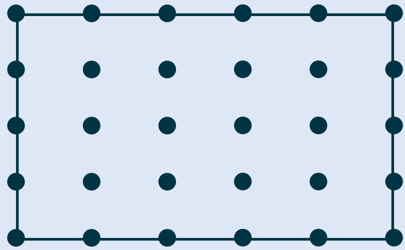


Make a polygon by inserting such pairs of pointers in the bolts. At each bolt, you should have two pointers.

NOTES

OBSERVATION

Let's verify pick's theorem for simple shapes such as a rectangle.



Area of this rectangle = 5 × 4

= 20 square units

Using Pick's Theorem, Area = i + b/2 - 1

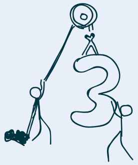
= 12 + 18/2 - 1

= 20 square units

You can also verify this for a combination of rectangles . Then for other shapes (for example, a triangle) for which you already know the formula for area. Interestingly, this works for all the polygons, given that the vertices lie on the grid.

EXPLORE

Pick’s Theorem gives the area of a 2D polygon. So a natural extension should be some theorem which gives volume of polyhedron. But sadly, there is no analogue of Pick’s theorem in three dimensions that expresses the volume of a polyhedron by counting its interior and boundary points.



$(a + b)^2 = a^2 + b^2 + 2ab$

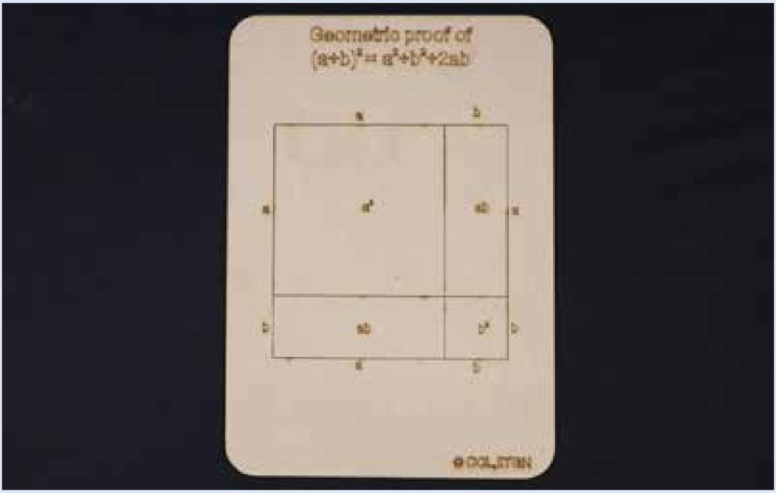
LEARNINGS

Algebra

Pythagoras Theorem

Area of a Square

Area of a Rectangle



In this activity, you can visually and geometrically see the algebraic formula. $(a + b)^2 = a^2 + b^2 + 2ab$.

WHAT TO DO?

1. Take out all the pieces from the sheet - two squares and two rectangles.
2. Now try to arrange them again inside the square.

WHAT'S GOING ON?

1. The area of a square is equal to $side^2$.
2. The sides of the big square (made using all the pieces) is a+b, making its area $(a+b)^2$.
3. Inside the big square, you have 2 rectangles of sides a and b, and two squares - one of side a and another of side b.
4. The area of the rectangles of sides a and b is ab.
5. Area of square with sides a is a^2 and that with sides b is b^2.

NOTES

6. Now, since the bigger square is made of two smaller squares and two rectangles, area of bigger square = area of two small squares + area of two rectangles

$$(a+b)^2 = a^2 + b^2 + ab + ab$$

$$\mathbf{(a+b)^2 = a^2 + b^2 + 2ab}$$

$(a - b)^2 = a^2 + b^2 - 2ab$

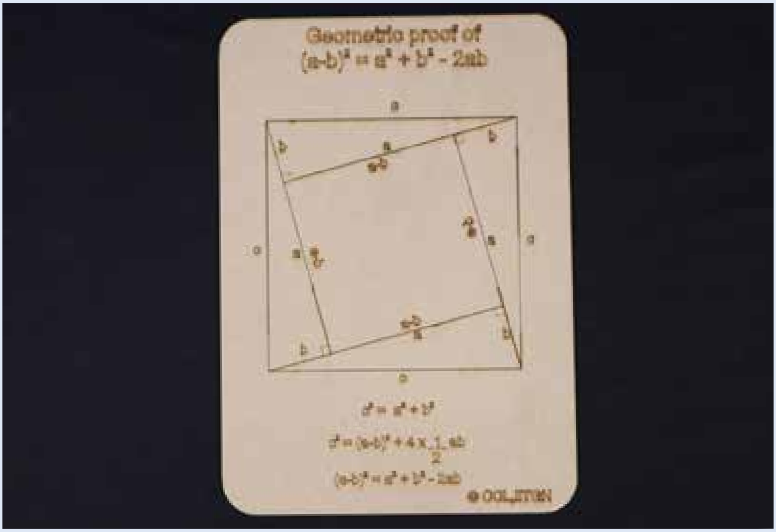
LEARNINGS

Algebra

Pythagoras Theorem

Area of a Square

Area of Right Angled Triangle



In this activity, you can visually and geometrically see the algebraic formula.
$(a - b)^2 = a^2 + b^2 - 2ab$.

WHAT TO DO?

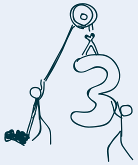
1. Take out all the pieces from the sheet- the four triangles and a square.
2. Now try to arrange them again inside the square, without looking at the answer (this is surprisingly challenging to do!)

NOTES

WHAT'S GOING ON?

1. The area of a square is equal to side^2.
2. So the area of the big square with sides c is equal to c^2. Similarly, the area of the smaller square with sides (a-b) is equal to $(a-b)^2$.
3. The area of a triangle is 1/2 x base x height. So the area of each of four right-angled triangles is 1/2 x a x b.
4. The total area of four such triangles is 4 x 1/2 x a x b = 2ab (you can also join two of these triangles and form a rectangle with sides a and b. The area of two such rectangles would again be 2ab).
5. As the triangles are right angled, you can also write using Pythagoras rule that $c^2 = a^2 + b^2$.
6. Now as the bigger square is made of one smaller square and the 4 triangles, area of bigger square = area of smaller square + area of 4 triangles

$$c^2 = (a-b)^2 + 2ab$$
$$a^2 + b^2 = (a-b)^2 + 2ab$$
$$\mathbf{(a-b)^2 = a^2+b^2 - 2ab}$$

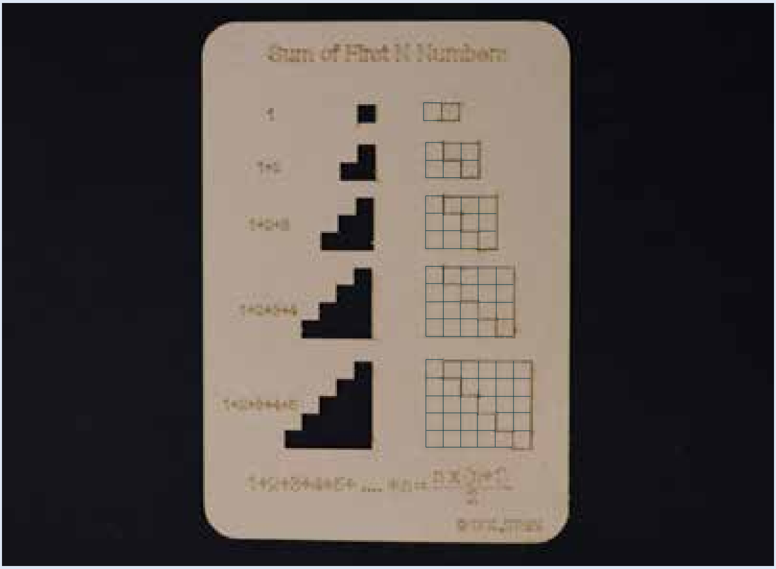


SUM OF FIRST N NUMBERS

LEARNINGS

Arithmetic Progression

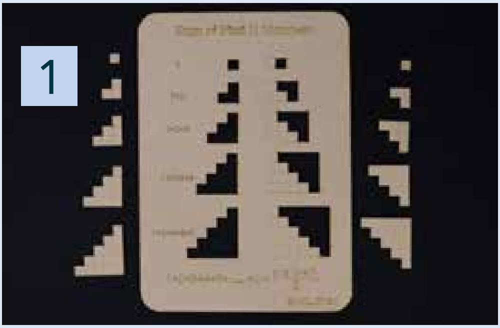
Area of a Rectangle



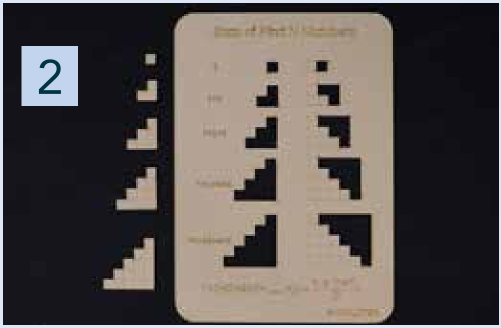
What is the sum of all numbers from 1 to 100, i.e., 1 + 2 + 3 + 4 + ... + 100 ? This same question was given to Carl Friedrich Gauss (a famous mathematician) by his teacher and he came up with an ingenious answer. You can also find the sum of natural numbers in this activity in a geometrical way!

WHAT TO DO?

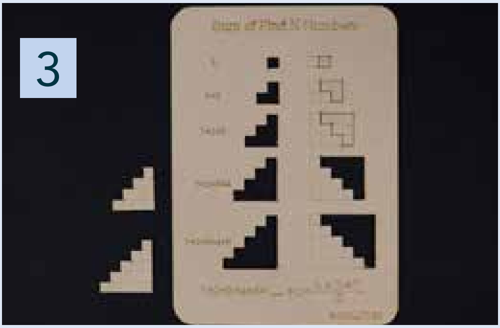
1. Remove all the unmarked pieces (where grid is not marked) from the right side of the sheet.
2. Take out the pieces from the left side and place them inside the hollow space at the right so that they form a rectangle.
3. Calculate the area of the rectangle formed by these two pieces.
4. Divide the area obtained by 2. This is equal to the sum of first N natural numbers.



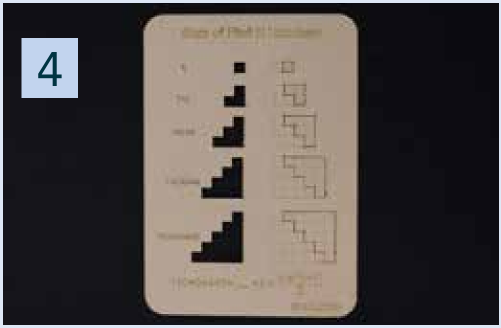
Remove all the pieces from the sheet.



You will only need the pieces shown above.



Place the pieces inside the hollow spaces on the right. Calculate the area of the rectangle formed after this.



Do this for all the pieces.

NOTES

OBSERVATION

1. All the numbers are represented by small squares. Note that all these small squares are of equal size. We would call them unit squares.
2. For example, the number 2 is represented by 2 unit squares, number 3 by 3 unit squares and so on.
3. If the area of the unit squares is assumed to be 1, the total area of the piece would be equal to the sum we want to calculate.
4. Two of these pieces always form a rectangle whose area can be easily calculated (area of rectangle = length × breadth)
5. And as the rectangle is made of two identical pieces, the area of one piece would be half of the area of the rectangle.
6. This would give us the sum of first N natural numbers.

WHAT'S GOING ON?

1. For the first two numbers (1 and 2), the rectangle formed would have the dimensions 2 × 3 and area would be 2 × 3 = 6 square units. The area of the individual pieces forming the rectangle would be half of the rectangle as they are identical. Therefore, we can say that the sum 1+2 = 3.
2. For the first three numbers (1, 2 and 3), the rectangle formed would have the dimensions 3 × 4 and its area would be 3 × 4 = 12 square units. As before, the sum 1+2+3 = 6.
3. You can also say that we can manually count the number of squares to find the sum, but that would be difficult to do for large numbers. Imagine counting total number of squares for 1 + 2 + 3 +....................... 100!
4. For first N numbers, the sides of the rectangle formed would be n and n+1. Area of this rectangle would therefore be n(n+1). as both pieces are same in area, the area of one of the pieces is n × (n+1)/2.
   Sum of first N natural numbers = n(n+1)/2.
5. Therefore the sum of numbers from 1 to 100 is (100 × 100 + 1)/2 = 5050.

NOTES

EXPLORE

1. The sequence 1, 2, 3, 4, 5 ,... is an example of Arithmetic sequence. It's a sequence where difference between two consecutive terms is same.
2. In this case, the difference is 1 for any two consecutive terms. You can have another sequence where the difference is some other number. For example, in 1, 4, 7, 10,...(and so on), the difference between consecutive terms is 3.

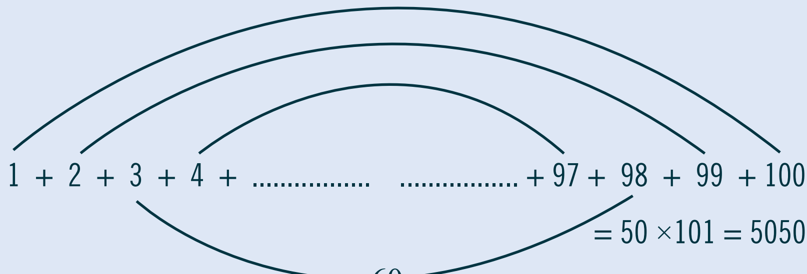
Story of Gauss

One day, a teacher wrote numbers from 1 to 100 and asked his students to calculate the sum of all the numbers. All the students started adding the numbers. Initially, the process was fast like an express train, but when they reached the middle numbers, it became a passenger train.

One boy was sitting at the back, starting at the board and doing nothing. The teacher got angry, and asked him why isn't is adding the numbers.The students remained quiet for some time, and then suddenly, said 5050.

He recognized that the first and last number added together give 101. The second and second last added together also give 101 and so on. And there are 50 such pairs with sum equal to 101. Therefore, the total sum is equal to $50 \times 101 = 5050$

This child, later became the most famous mathematician, **Carl Friedrich Gauss**. This story depicts that math is not just about formulas, but interesting patterns!

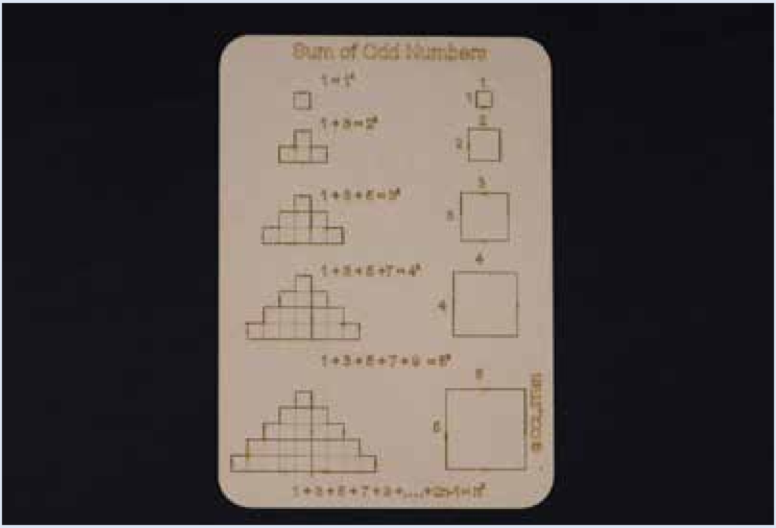


SUM OF ODD NUMBERS

LEARNINGS

Arithmetic Progression

Area of a Square

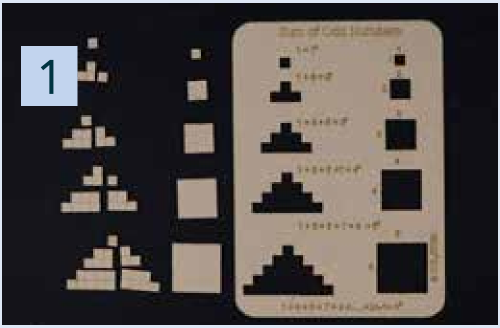


What is 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 ? You'd probably start adding the numbers one by one, but you can find a faster way if you play with the puzzle pieces for a while.

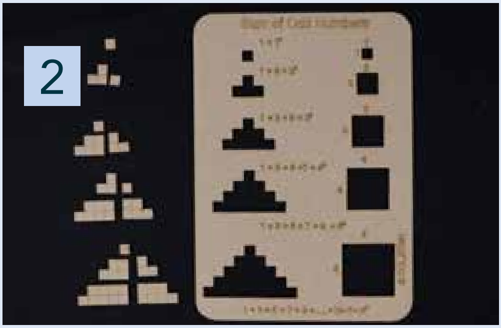
This activity does just that - showcasing an algebraic equation using a jigsaw puzzle!

WHAT TO DO?

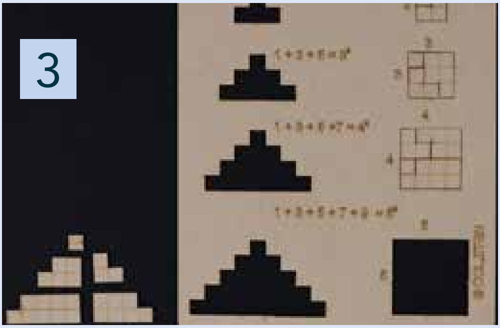
1. Remove all the squares from the sheet given at the right side.
2. Remove the pieces at the left and try to fit them into the corresponding squares at their right.
3. You would find that all the pieces fit exactly inside the squares.



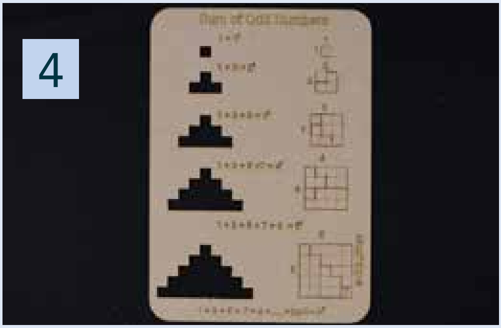
Remove the shown pieces from the sheet, and keep the squares aside.



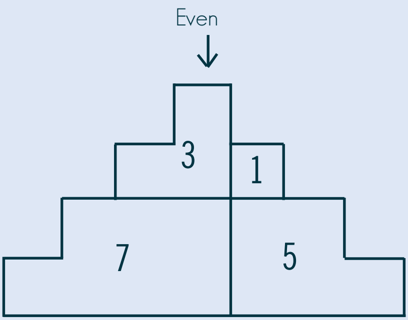
The pieces on the left are the pieces that you will need.



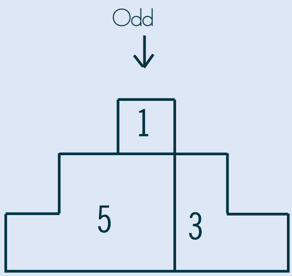
Fit these pieces in the corresponding squares on the right.



You would find that all the pieces fit exactly inside the squares.



and so on...



and so on...

NOTES

OBSERVATION

1. All the numbers are represented by small squares. Note that all these small squares are of equal size. We would call them unit squares.
2. For example, the number 3 is represented by 3 unit squares, number 5 by 5 unit squares and so on.
3. Only the odd numbers are shown - numbers which are indivisible by 2 (1, 3, 5, 7, 9 ,.. and so on).
4. The pieces representing the numbers fit in the respective squares at the right side of the sheet.
5. This means that the sum of the sequence 1, 3, 5, 7, 9,... is always a perfect square. You can add more numbers in the sequence and try for yourself.

WHAT'S GOING ON?

1. Let's add some odd numbers and see how this puzzle helps us find the sum.
2. For the first two odd numbers (1 and 3), you have 1 square on the top and 3 more beneath it. Now rearrange these pieces in a square of side 2 (at the right side of the sheet). You'll see that they fit perfectly in this square.
3. For the first three odd numbers (1, 3 and 5), you can fit them in a square of side 3 having area equal to 9 square units.
4. Try this with the next two cases in the sheet and you would find that the pieces fit exactly in the squares at the right.
5. If you are adding 1 + 3 + 5 + 7 + 9 (number of terms: 5), the sum comes out to be 25 (5^2).
6. This can be generalized for any number of terms. If the number of terms is 6 (which means you have to calculate1 +3 + 5+ 7 + 9 +11), their sum would come out to be 36 (6^2).
7. Therefore for n number of terms, the sum would be n^2.

EXPLORE

Try to find the sum of numbers 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 using this information.

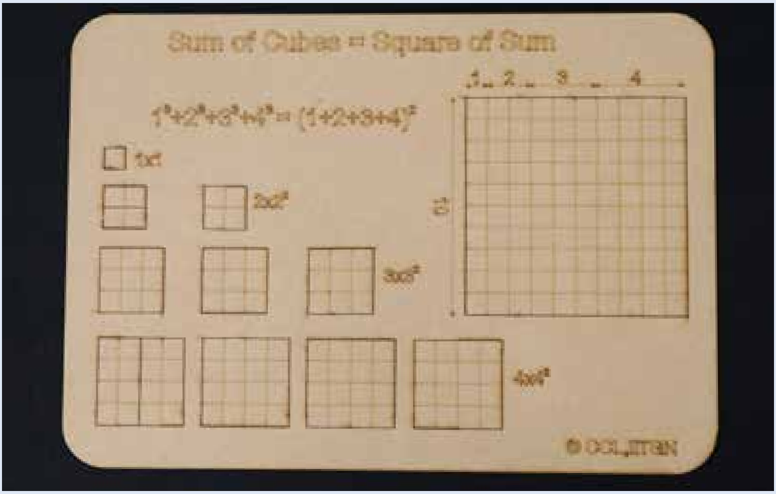
SUM OF CUBES

LEARNINGS

Area of a Square

Relation Between Cube and Square

Progressions

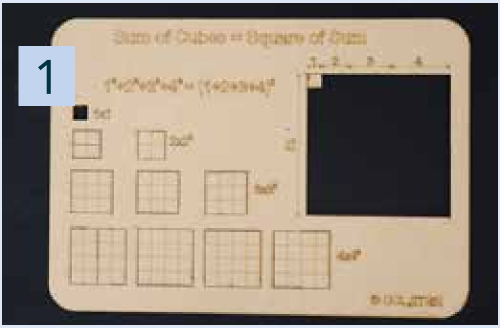


The sum of cubes of first n natural numbers is equal to square of their sum. With this interesting puzzle game, you can see this relation of cubes and squares in a geometrical way.

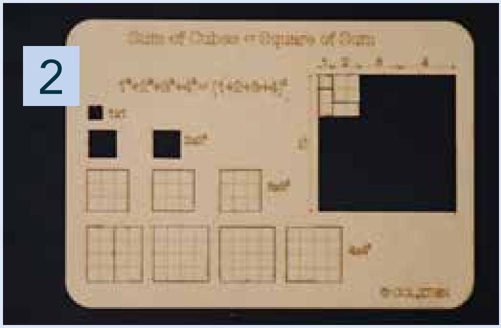
$$1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2$$
$$100 = 100$$

WHAT TO DO?

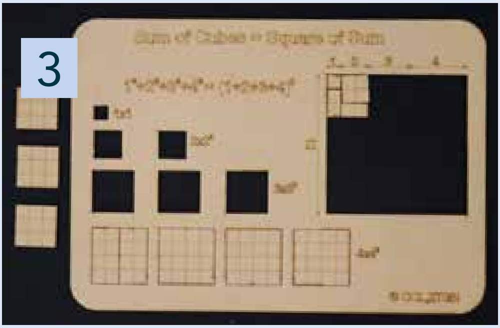
1. Remove the big 10 × 10 square given at the right side of the sheet.
2. Remove the 1 × 1 square and two 2 × 2 squares from the sheet. Fit all these pieces inside a square of 3 (1 + 2) units in the empty square at the right.
3. Now remove the three 3 × 3 squares also and fit them (along with the pieces already placed) inside a square of 6 (1 + 2 + 3) units.
4. Finally, remove the four 4 × 4 squares and fit them inside the square of 10 (1 + 2 + 3 + 4) units.



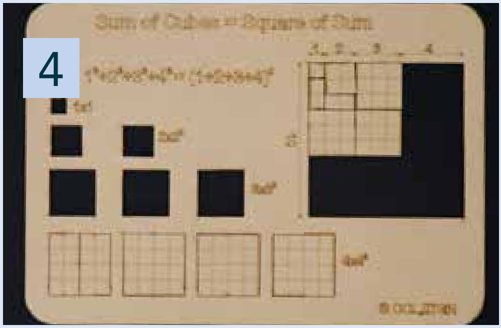
Put the 1x1 square from the sheet in 10 x10 square.



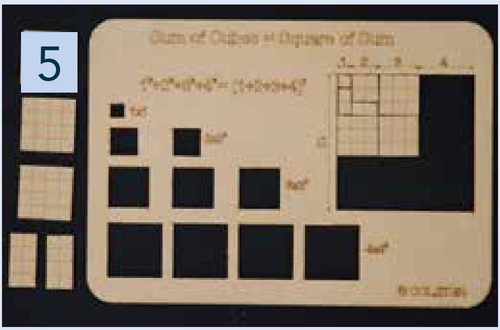
Remove the two 2x2 squares from the sheet and fit all these pieces inside a square of 3 (=1+2) units.



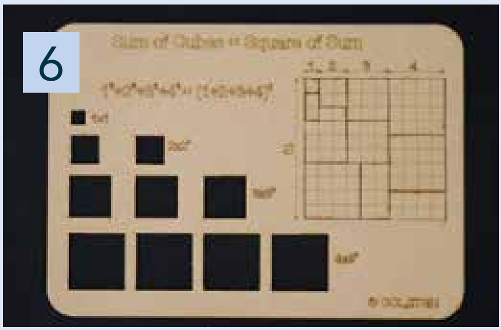
Now remove the three 3x3 squares from the sheet.



Fit them (along with the pieces already placed inside) inside a square of 6 (= 1+2+3) units.



Finally, remove the four 4 x 4 squares and fit them inside the square of 10 (=1+2+3+4) units.



All 1^2, 2^2, 3^2 and 4^2 pieces are empty and fitted in 10x10 square.

NOTES

OBSERVATION

1. All the numbers are represented by small squares. Note that all these small squares are of equal size. We would call them unit squares.
2. For example, the number 3 is represented by 3 unit squares, number 5 by 5 unit squares and so on.
3. The squares of 1×1 and 2×2 completely fit inside a square of 3 square units. You will notice something similar when you try to fit 3×3 squares (they would fit inside the a square of 6 units: $1 + 2 + 3$) and so on.
4. All the smaller squares would eventually fit inside the bigger 10×10 square $(1 + 2 + 3 + 4 = 10)$.

WHAT'S GOING ON?

1. First, we have to understand how to geometrically represent the cube of any number.
2. Consider 2^3 i.e.$2\times 2 \times 2$. Now consider a square of side 2. Its area will be 2×2. Now if you have two such squares the total area would be $2 \times (2 \times 2)$ i.e. 2^3.
3. Similarly 3^3 means area of three squares of dimension 3×3 and so on.
4. There is one square of side 1, two of side 2, three of side 3, and four of side 4 in the sheet. This way, we have geometrically represented the cubes of 1, 2, 3 and 4.
5. The sum of these numbers $(1 + 2 + 3 + 4)$ is 10 and all the smaller squares fit inside the square of side 10 .

   Sum of cubes = square of sum

EXPLORE

1. The sum of first N natural numbers is n(n+1)/2.

   Therefore, we can also write the formula as: $1^3 + 2^3 + 3^3 +....+ n^3 = (1 + 2 + 3 +......n)^2 = (n(n+1)/2)^2$
2. In this activity, the cubes of numbers were represented in terms of the area of squares. 2^3 was written as 2×2^2, which means that the total area of two 2×2 squares would be equal to 2^3.
3. This can also be extended to 3D and volumes. For example, 2^4 can be written as 2×2^3 which means that the total volume of two $2 \times 2 \times 2$ cubes would be equal to 2^4.

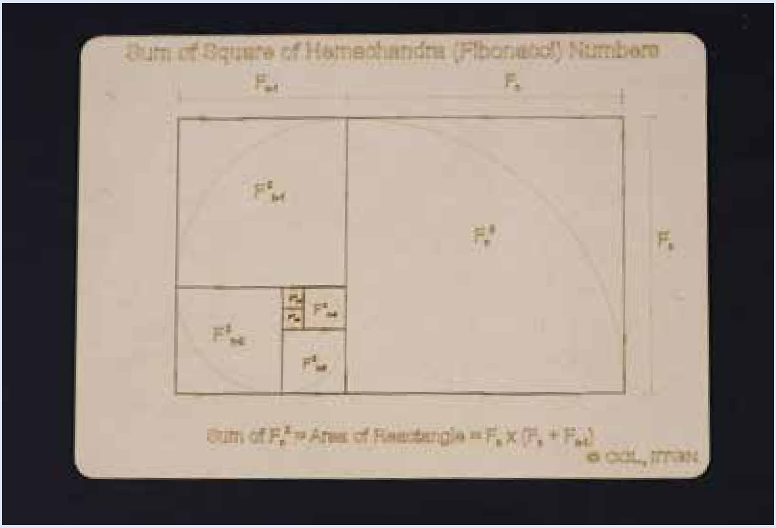
SUM OF SQUARE OF HEMCHANDRA NUMBERS

LEARNINGS

Hemachandra Series

Fibonacci Series

Area of a Rectangle



Hemachandra Series (or Fibonacci Series) is obtained by adding the last two terms to get the next term. So the series goes like - 1, 1, 2, 3, 5, 8, 13, 21, 34,.... and so on. Now can you calculate the sum of square of these terms?

One method is to just square the numbers and then add all of them. This activity provides a faster, geometric alternative to do this.

WHAT TO DO?

1. Take out all the square pieces from the sheet marked as 1, 1, 2, 3, 5, 8 and 13.
2. Now rearrange them again inside the square, (without looking at the answer).

NOTES

OBSERVATION

1. The Fibonacci series is such that each term is the sum of two preceding terms. $F_n = F_{n-1} + F_{n-2}$.
2. $F_1 = F_2 = 1$; $F_3 = F_1 + F_2 = 2$; $F_4 = F_2 + F_3 = 3$ and so on. The series is therefore 1, 1, 2, 3, 5, 8, 13, 21, 34 and so on.
3. The sides of the square are 1 cm, 2 cm, 3 cm, 5 cm, 8 cm and 13 cm.
4. The length of the sides are such that they are in Fibonacci series.
5. The area of the square pieces represent the squares of the Fibonacci numbers.
6. The total area of all the squares is equal to the sum of squares of Fibonacci numbers.
7. The total area is equal to the area of rectangle with breadth 13 cm and length 13 + 8 = 21 cm. Area = 13 x 21 = 273 cm^2

WHAT'S GOING ON?

1. We can find the sum of squares Fibonacci numbers by multiplying the last number (not the square of the last number, but the last number itself) and the addition of last and second last number. In other words,
2. Sum of squares of Fibonacci numbers = $F_n \times (F_n + F_{n-1})$
3. Therefore, Sum of squares of Fibonacci numbers $= \sum F_{n^2} = F_n \times (F_n + F_{n-1})$
4. For example, the series is 1, 1, 2, 3, 5, 8, 13. The square of terms are 1, 1, 4, 9, 25 ,64, 169. Sum of all these numbers is 1 + 1 + 4 + 9 + 25 + 64 + 169 = 273 which can also be calculated as 13 x (13+8) = 273.

EXPLORE

1. The ratio between two consecutive Fibonacci numbers converges (or tends) to Golden Ratio which is approximately equal to 1.618.
2. There is another sequence called the Tribonacci numbers, where each term is equal to the sum of three preceding terms. $F_n = F_{n-1} + F_{n-2} + F_{n-3}$

SUM OF GP (SUM = 1)

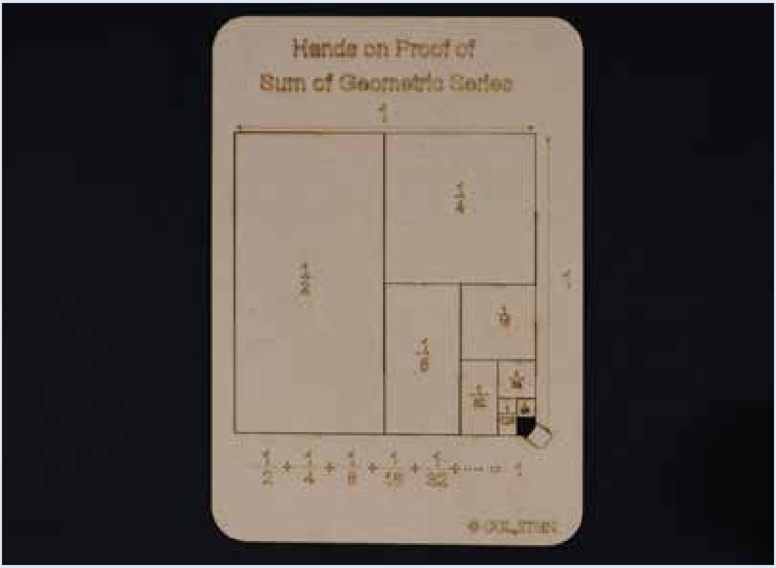
LEARNINGS

Geometric Series

Area of a Rectangle

Convergent Series

Divergent Series



Can you add a series that has infinite number of terms? Would the answer still be a finite number? Let's find out in this activity.

WHAT TO DO?

1. Take out all the pieces from the square marked as 1/2, 1/4, 1/8, and so on.
2. Now rearrange them again inside the square (without looking at the picture).

NOTES

OBSERVATION

1. The side of the big square is taken as 1 unit. Therefore the area of the square is 1 square units.
2. If you divide the square in half, the area of that piece would be 1/2.
3. If you further divide that part in two equal parts, the area would be 1/4 and so on. The area of the smaller pieces eventually becomes smaller and smaller and converges to 0.
4. All the pieces together make up the big square.
5. Therefore, 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + ... = 1

WHAT'S GOING ON?

1. The sequence 1/2 ,1/4, 1/8,..... is an example of Geometric series. It's a series where ratio of two consecutive terms is same.
2. In this case, the ratio is 1/2 for any two consecutive terms. You can have another sequence where the difference is some other number.
3. For example, in 1, 4, 16, 64,...(and so on), the difference between consecutive terms is 4.
4. If this common ratio is less than1, the terms keep getting smaller and smaller eventually become zero. This is an example of convergent series.
5. If this ratio is more than 1, the sum of infinite terms won't be a finite number. This is an example of divergent series.

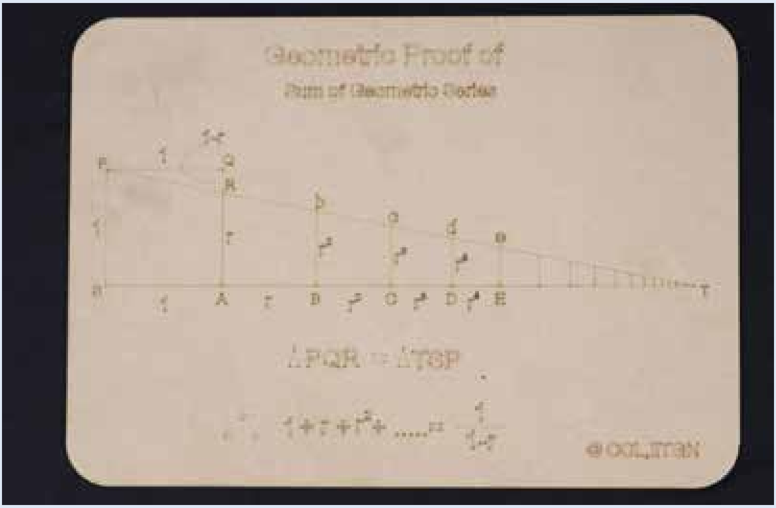
SUM OF GENERAL INFINITE GP

LEARNINGS

Geometric Progression

Basic Proportionality Theorem

Convergent Series



Can you add a series that has infinite number of terms? Would the answer still be a finite number? Let's find out with this sheet.

WHAT TO DO?

To construct the given triangle:

1. Make a square PSAQ of unit length (length = 1).
2. Mark length AR = r, which is the common ratio of the geometric progression. The remaining length RQ = 1 - r
3. Extend the line segments SA and PR. Mark the point of intersection at T.
4. Mark the length AB = r and draw a perpendicular line segment such that Bb = r^2 (the point b would automatically lie on PT. Why?)
5. In the same way, mark the lengths BC, CD and so on. The successive lengths would be r^3, r^4, r^5 and so on.

NOTES

WHAT'S GOING ON?

1. As r is less than 1, as you increase the powers of r, you will get smaller and smaller numbers and eventually, you will reach 0 (after infinite terms).
2. Thus, the line segments PS, RA, Bb, Cc, Dd, Ee... will eventually become smaller and smaller and will converge to a point.
3. ∠PQR =∠TSP (right angles), ∠QPR = ∠STP (alternate angles). The third angles are also equal (as the total angle of a triangle is 180° and if two angles are equal, the third have to be equal). Therefore, ΔPQR ~ ΔTSP (AAA similarity).

   TS/PQ = PS/QR.
   $(1 + r + r^2 + ...) / 1 = 1/1-r$
   $1 + r + r^2 + ...\ = 1/(1-r)$

EXPLORE

What if you had chosen SA = 1, BA = r and say CB = r^3 instead of r^2? Will the method also work in that case? Why or Why not?

MISSING AREA PUZZLE

LEARNINGS

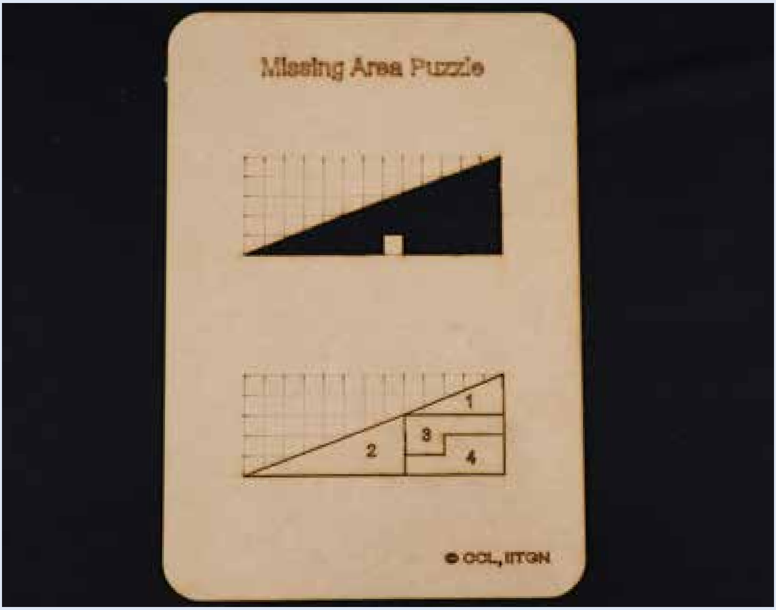
Straight line

Slope

Area of a Triangle

Area of Rectangle

Right Angle Triangle



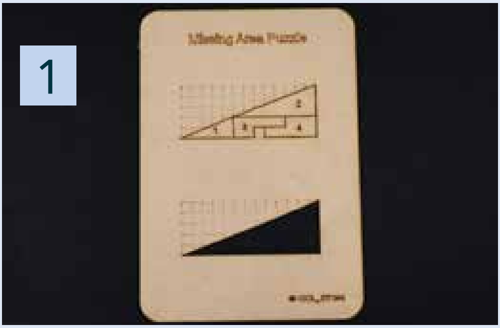
The puzzle pieces leave a square block in the above triangle but fit exactly inside the triangle below, even thought both the triangles are same. Try out this activity involving shapes and test your detective skills. Figure out where did the area go missing from one shape to the other!

WHAT TO DO?

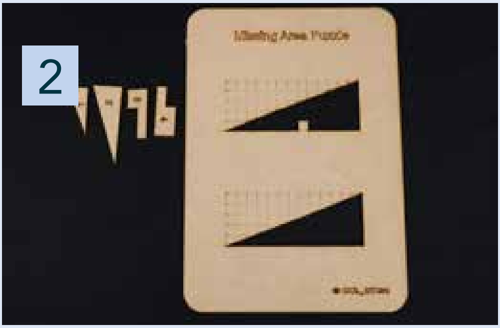
1. Take out the pieces of the triangle (marked as 1, 2, 3, and 4) at the top of the sheet.

Remove the triangle from the bottom of the sheet. Both the triangles look exactly the same - both of them are in 13 × 5 cm grid.

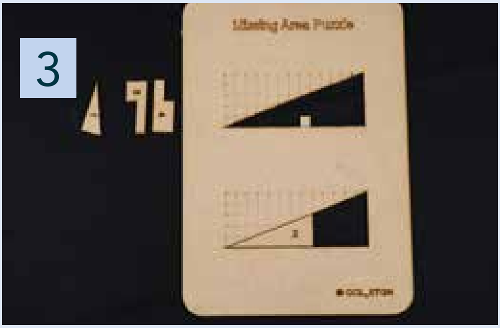
2. Fit the pieces of the triangle (marked as 1, 2, 3, 4) in the hollow triangle.
3. You will find that the pieces completely fill the triangle, whereas in earlier configuration, one unit square was left with the same pieces!



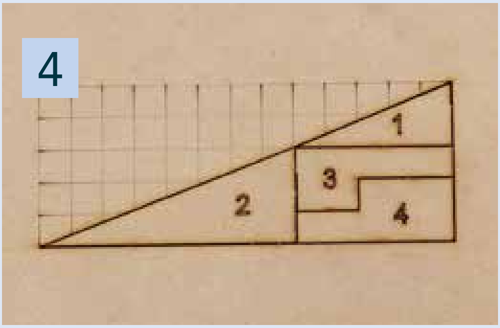
Remove the triangle at the bottom



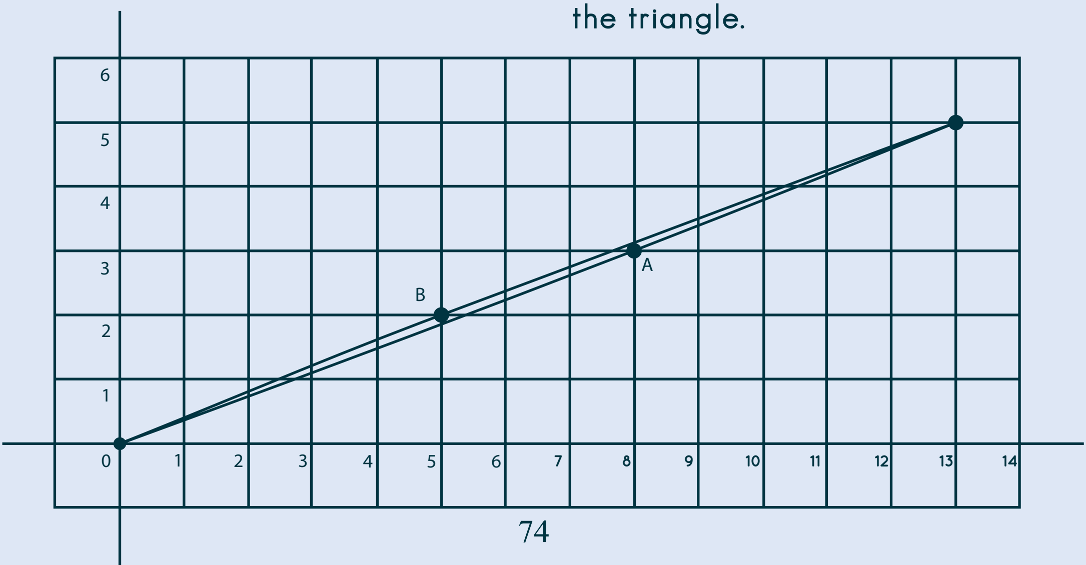
Take out the pieces of the triangle (marked as 1, 2, 3, and 4) at the top of the sheet.



Fit the pieces of the triangle marked as 2 in the triangle below, as shown.



Fit these pieces of the triangle in the hollow triangle below. You will find that the pieces completely fill the triangle.



NOTES

OBSERVATION

1. If you look carefully, you would find that the hypotenuse of the triangles is not really a straight line.
2. The slopes of pieces 1 and 2 are different but isn't easily recognized by our eyes. The slope of the piece marked as 1 is 8/3 (≈2.667) and that of piece 2 is 5/2 (2.5).
3. Overlaying the hypotenuses from both figures results in a very thin parallelogram with an area of exactly one grid square, hence the “missing” area.

   Eureka! We have solved the missing area case.

EXPLORE

This missing area puzzle, and many other optical illusions depict that it is important to also look at textual descriptions rather than just at figures shapes. Sometimes the human eye is susceptible to errors which can be avoided if we also process other information about the situation.

MAGICAL RATIO OF A4

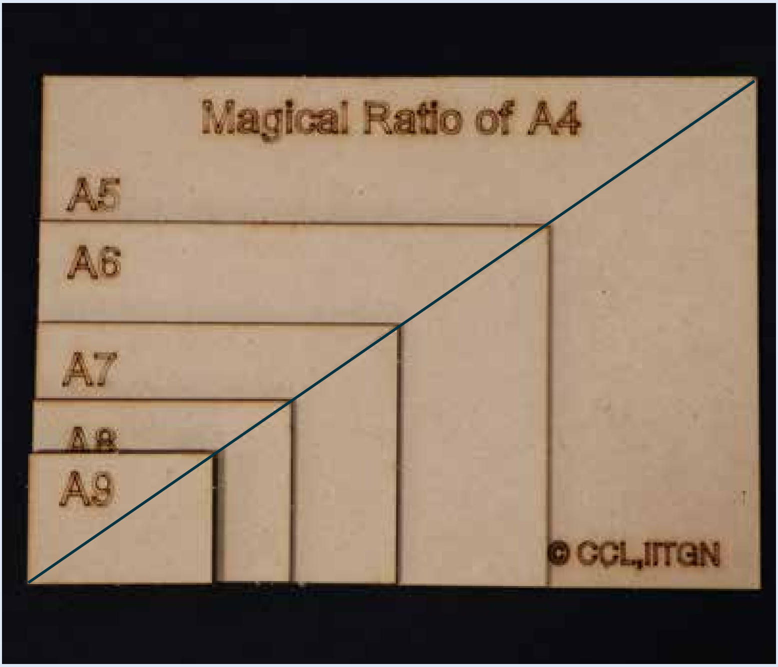
LEARNINGS

Aspect Ratio

A4 sheet

Slope

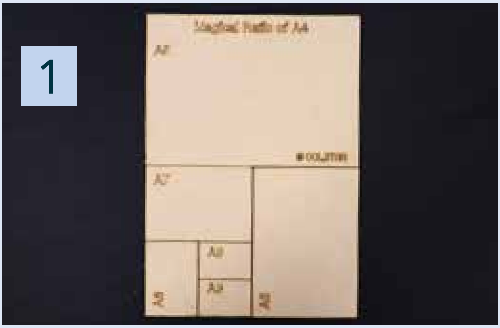
Magnification



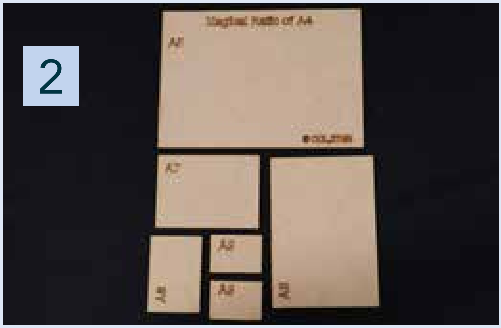
We see and use A4 sheets almost daily. We can see that it is a rectangle, but is there something special about this rectangle? What is the ratio of its length and breadth? Let's find out in this activity.

WHAT TO DO?

1. Take out the parts marked as A5, A6, A7 and A8 from the sheet.
2. Put all the parts - A5, A6, A7, A8 on top of each other, with A5 sheet at the bottom.
3. You will see that the diagonals of all the parts will coincide with each other.

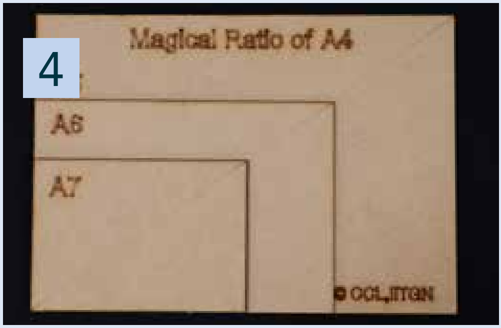


The initial sheet will look like this

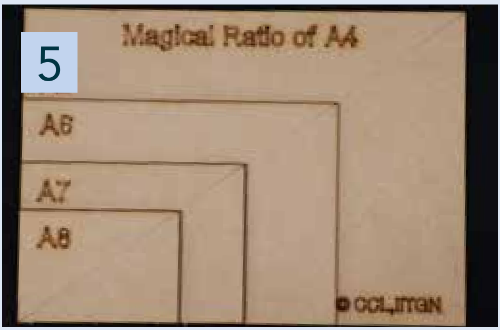


Break up the pieces marked as A5, A6, A7 and A8 from the sheet

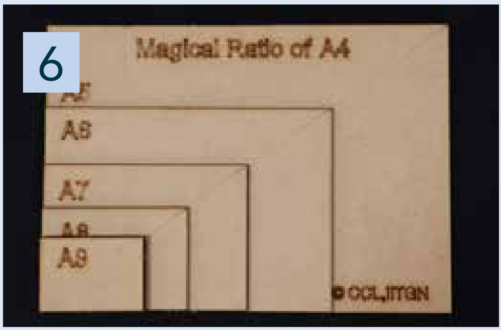
Place the piece marked A6 on the piece marked A5, such that the diagonal line matches.



Next place the piece marked A7 on the piece marked A6, such that the diagonal line matches



Place the piece marked A8 on the piece marked A7, such that the diagonal line matches.



You will see that the diagonals of all the parts will coincide with each other.

NOTES

OBSERVATION

1. If the diagonals are coinciding with each other, it means that the ratio of length and breadth of all these rectangles is same (why?).
2. Let's assume that the length and bredth of A4 sheet is r and 1 respectively.
3. When the length is halved, the longer side becomes 1 and the smaller side is r/2. So the ratio is 1/(r/2). And we are claiming that these two ratios are equal.

   So $r/1 = 1/(r/2)$
   Solving it gives, $r = \sqrt{2}$

4. Therefore the ratio of length and breadth of A4, A5, A6,... is $\sqrt{2}$.

WHAT'S GOING ON?

1. When you keep two A4 sheets side by side, You get an A3 sheet. With two A3 sheets, you get A2 and so on.
2. Similarly, if you go in other direction and halve the A4 sheet, you get an A5 sheet. The specialty of this A-series is that the ratio or proportion of all the sheets (A0, A1, A2, A3, A4,...) is exactly the same.
3. This same ratio implies that the photo or the text to be printed on the paper doesn't get stretched or squeezed, no matter the paper we choose for printing (A3, A4, A5,...).
4. So if you want to change the type of sheet,you don't have to adjust the aspect ratio. It is as if the same sheet has been zoomed or magnified!

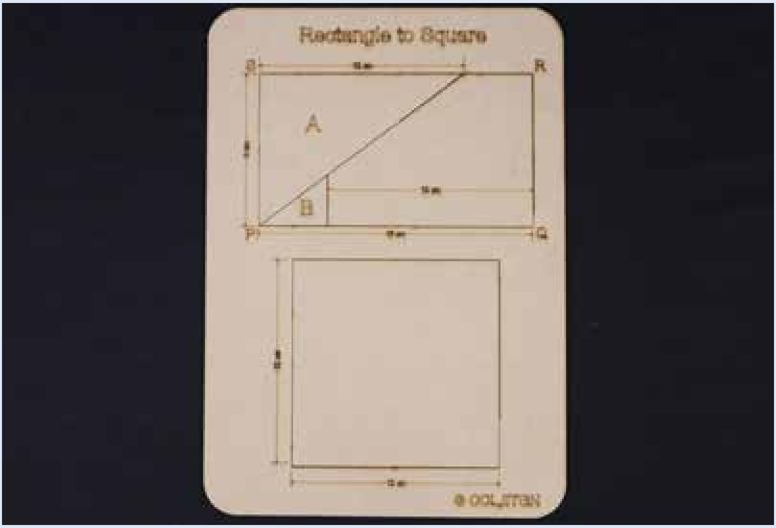
RECTANGLE TO SQUARE (SLIDING METHOD)

LEARNINGS

Mensuration

Area of a Rectangle

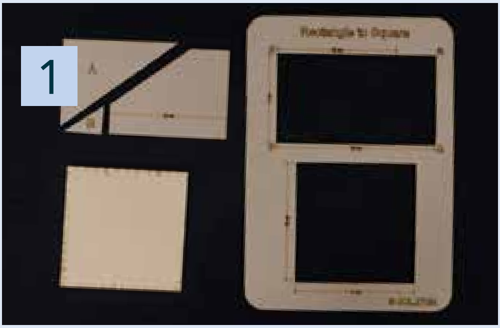
Area of a Square



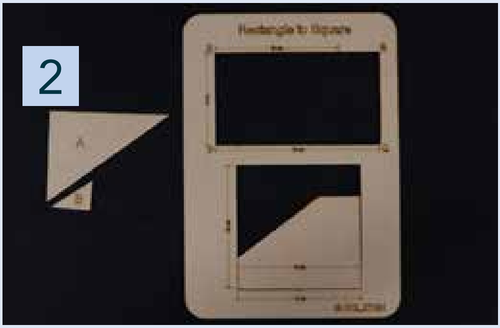
Can you make a square out of a rectangle, having the same area as the starting rectangle? This means that you can't throw any piece! Learn how to convert a rectangle to a square in this activity!

WHAT TO DO?

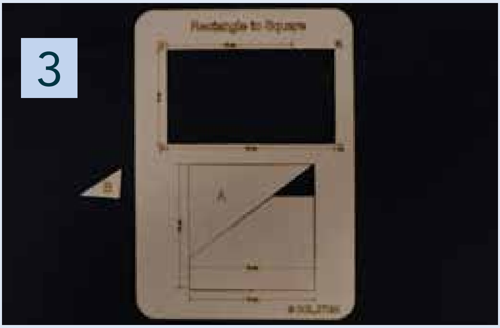
1. Remove the square at the bottom of the sheet.
2. Now take out the pieces of the rectangle and fit them inside the square



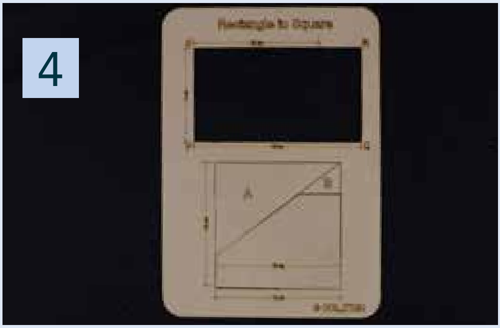
Take out the pieces from the module, keep the square aside.



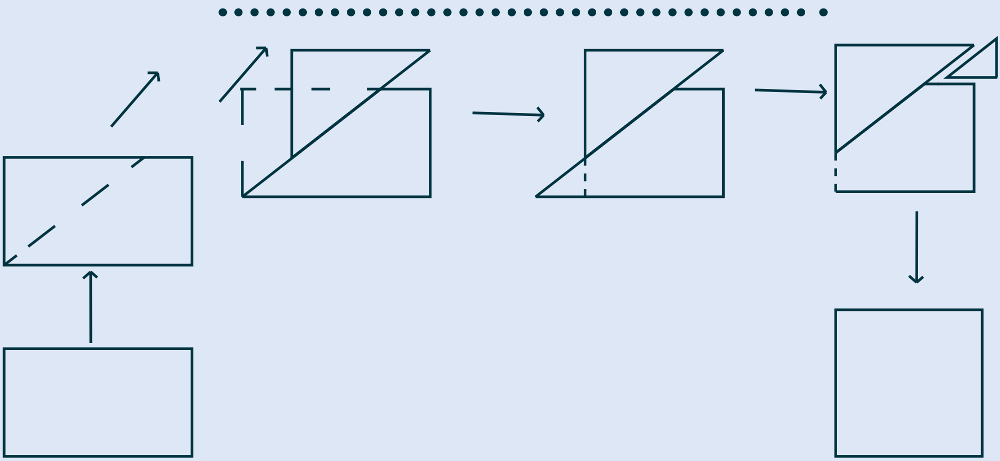
Start placing the pieces from the rectangle into the square below as shown.



Add the next piece in the square.



Complete the entire square, make a note of the sides of the square.



NOTES

OBSERVATION

1. The sides of the rectangle are 16 cm × 9 cm. Therefore, the area of the rectangle is 16 x 9 = 144 cm^2.
2. As you have to make the square using all the rectangle (we can't throw any piece), the area of rectangle and square has to be equal.
3. So the area of the required square is 144 cm^2 and the side should be $\sqrt{144}$ = 12 cm.

WHAT'S GOING ON?

1. Let's see how to cut the rectangle and convert it into a square. Use a paper of the same dimensions (16cm × 9cm) to understand the cuts.
2. The side of the required square is 12 cm. Mark this length on the rectangle (ST) and make the first cut PT.
3. Slide the piece A such that point T is in line with the side RQ of the rectangle.
4. Cut the triangle B and place it in the empty space below point T.
5. The square obtained after this would have the dimensions 12 cm × 12 cm.

EXPLORE

1. Make another rectangle using cardboard or paper having dimensions 25 cm × 9 cm and convert it into a square. The sides of the square would be 15 cm (convince yourself!)
2. Can you use this method to convert any rectangle to a square? What is the maximum ratio of length and bredth of the rectangle for which it would work?

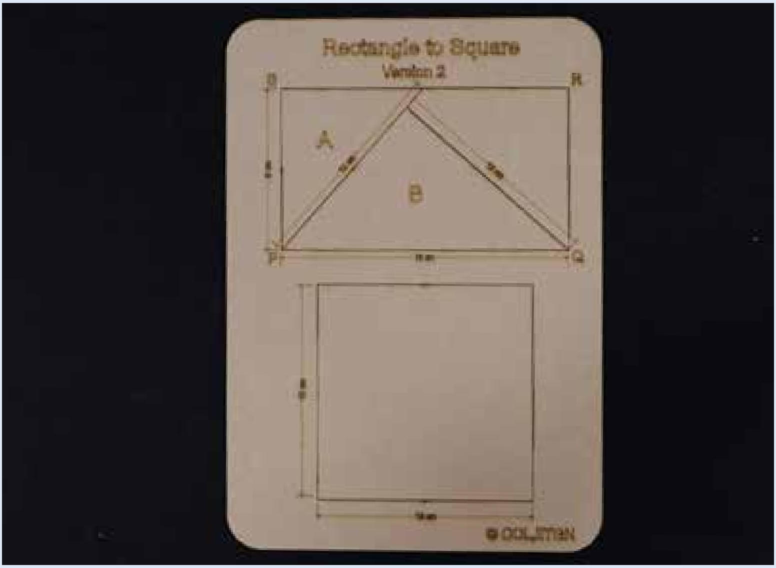
RECTANGLE TO SQUARE (PARALLELOGRAM METHOD)

LEARNINGS

Mensuration

Area of a Rectangle

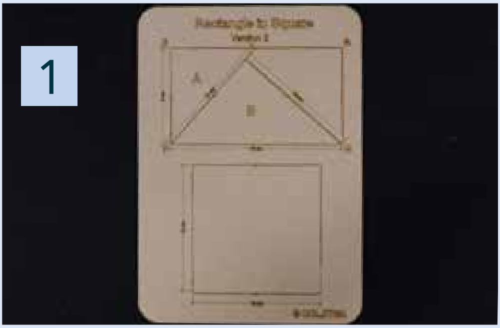
Area of Square



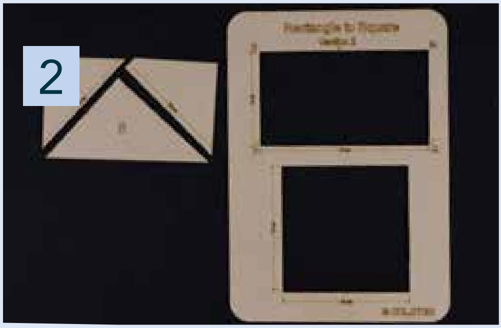
Can you make a square out of a rectangle, having the same area as the starting rectangle? This means that you can't throw any piece! Learn how to convert a rectangle to a square in this activity!

WHAT TO DO?

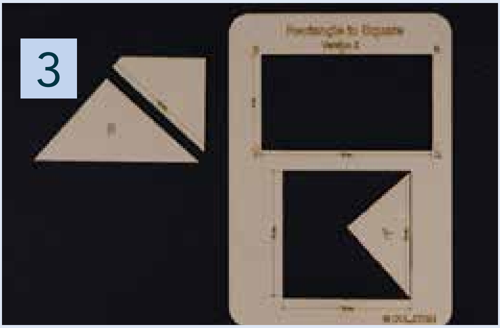
1. Remove the square at the bottom of the sheet.
2. Now take out the pieces of the rectangle and fit them inside the square.



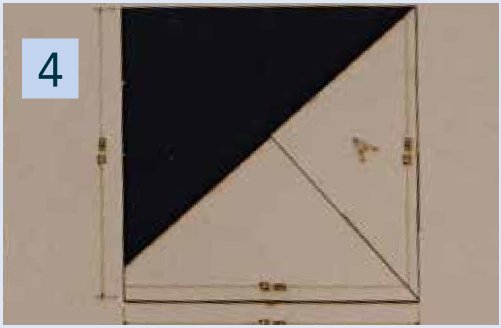
Take out the pieces from the module, keep the square aside.



Start placing the pieces from the rectangle into the square below as shown.

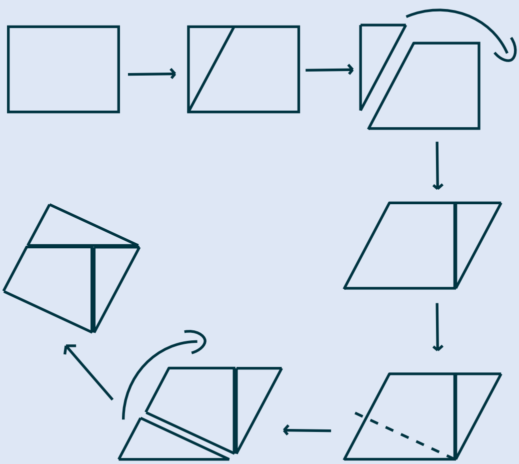


Add the next piece in the square.



Attach the piece shown next.

Complete the entire square, make a note of the sides of the square.



NOTES

OBSERVATION

1. The sides of the rectangle are 16 cm × 9 cm. Therefore, the area of the rectangle is 16 × 9 = 144 cm^2.
2. As you have to make the square using all the rectangle (you can't throw any piece), the area of rectangle and square has to be equal.
3. So the area of the required square is 144 cm^2 and the side should be $\sqrt{(144)}$ = 12 cm.

WHAT'S GOING ON?

1. Let's see how to cut the rectangle and convert it into a square.
2. The side of the required square is 12 cm. Mark this length of 12 cm on the rectangle (PT) and make the first cut PT.
3. Slide the piece A towards the side RQ. You now have a parallelogram whose two sides are equal to the sides of the required square.
4. From point Q, again measure 12 cm and make the cut QU.
5. Transfer this piece (marked as B) to the opposite side.
6. The square obtained after this would have the dimensions 12 cm × 12 cm.

EXPLORE

1. Using this method, you can convert other rectangles into squares. Make another rectangle using cardboard or paper having dimensions 25 cm × 9 cm and convert it into a square. The sides of the square would be 15 cm (convince yourself!)
2. Not just rectangles and squares, this method can actually be used to convert any parallelogram into any other parallelogram of same area!
3. Can you use this method to convert any rectangle to a square? What is the maximum ratio of length and bredth of the rectangle for which it would work?

EQUILATERAL TRIANGLE FROM RECTANGLE

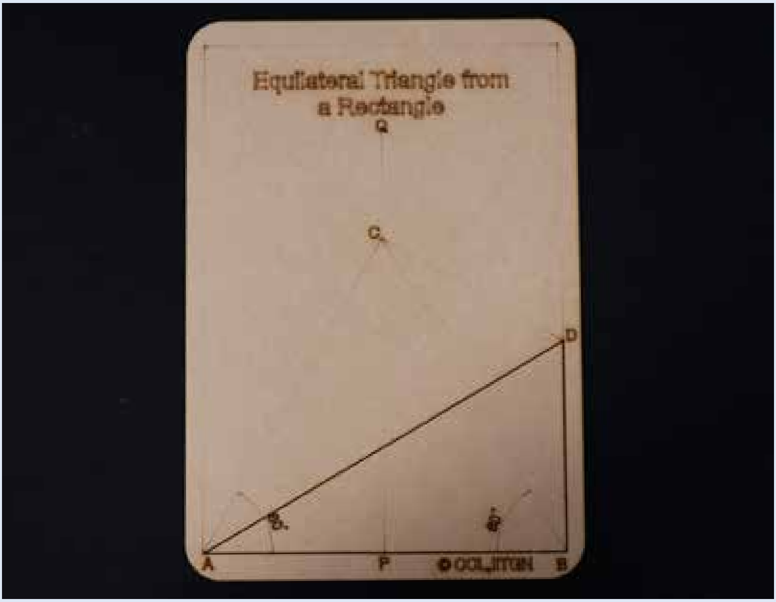
LEARNINGS

Mensuration

Equilateral Triangle

Perpendicular Bisector

Trisecting an Angle



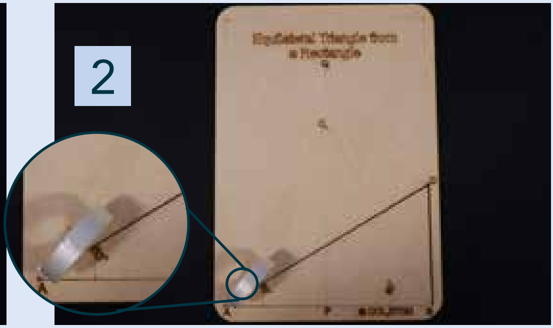
An equilateral triangle is a triangle in which all three sides are equal. But how do you make an equilateral triangle from a rectangle without measurement?

In this activity you can convert the given rectangle into an equilateral triangle by using an interesting property of perpendicular bisectors.

WHAT TO DO?

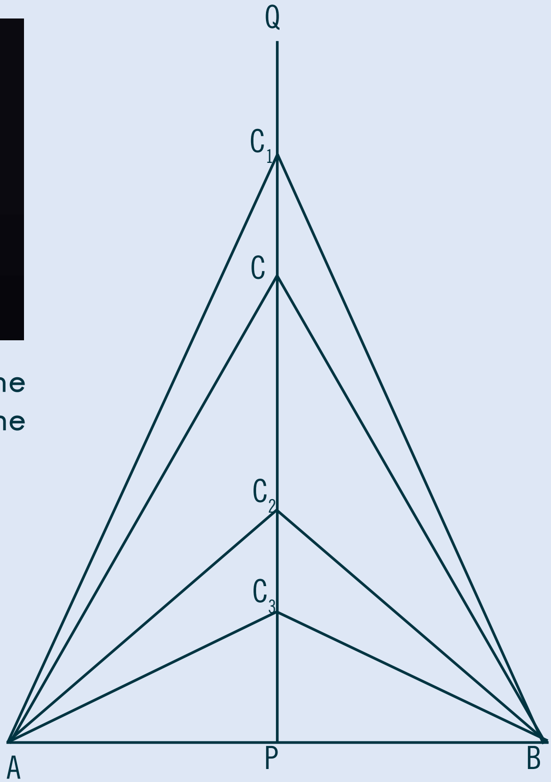
1. Put a small sticky tape in the middle of the line segment AD and fold the triangle ABD around AD.
2. The point B would intersect the line PQ at point C.
3. ΔABC is the required triangle.

Separate the triangle from the sheet as shown.



Put a small sticky tape on the line segment AD and fold the triangle ABD around AD.

The point B would intersect the line PQ at point C. Triangle ABC is the required triangle



Various triangles where the two sides are equal.

NOTES

WHAT'S GOING ON?

1. The line PQ shown in the middle of the rectangle is the perpendicular bisector of side AB.
2. An interesting property of perpendicular bisector is that all the points on it are equidistant from the ends of the line segment bisected. In this case, any point on PQ is equidistant from A and B.
3. In fact, all the triangles you get with these three points- A, B and one point on the perpendicular bisector- are at least isosceles.
4. Now all you have to do is to find a point C on the perpendicular bisector line so that the two lines drawn from it are equal to the base. Then our isosceles triangle would become equilateral.
5. When you lift the base AB and put it on the perpendicular bisector, this ensures that CA = AB. And CA = CB as the point lies on the perpendicular bisector.
6. Therefore all the sides are equal in length. Hence, this triangle ABC is an equilateral triangle!

EXPLORE

1. One more way is to use the fact that all angles of an equilateral triangle are 60°.
2. But how can you fold an angle of 60°? One way is to divide 90° angle in 3 equal parts and then taking two of those parts. This requires a little trial and error as you can't exactly trisect an angle.
3. As the angles of a rectangle are 90°, trisecting them would give angles of 60°.
4. Therefore, if you make angles of 60° at A and B, they would intersect at C and the third angle would also be 60° (if the two angles of a triangle are 60°, the third one has to be 60° as the sum of all angles of a triangle is always 180°).
5. The triangle ABC would therefore have all angles equal to 60° and would be an equilateral triangle.
6. Find all the lengths and angles marked in the sheet. Assume that the sides of the rectangle are 1 and $\sqrt{2}$ units.

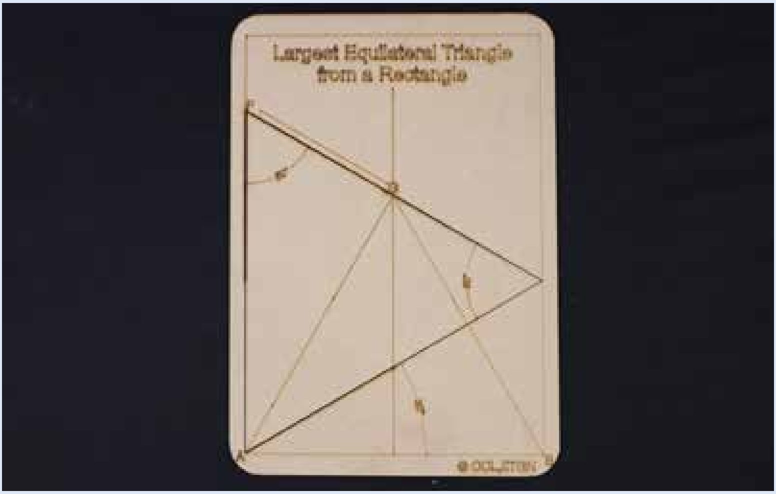
LARGEST EQUILATERAL TRIANGLE FROM RECTANGLE

LEARNINGS

Mensuration

Area of a triangle

Equilateral Triangle



What is the largest equilateral triangle you can make from a rectangle? Can it be equal to the length of the rectangle?
In this activity you would find the largest equilateral triangle in a rectangle.

WHAT TO DO?

1. Δ ABC is an equilateral triangle whose sides are equal to the breadth of the rectangle.
2. Imagine rotating the ΔABC about point A and extending the sides. You would see that the sides of the equilateral triangle would be more than AB.
3. If you rotate the original triangle by 30°, the edge of the triangle would coincide with that of the rectangle. This means that any more rotation would cause the third vertex to lie outside the rectangle.
4. Therefore, ΔAB′C′ is the largest equilateral triangle you can make in the rectangle.

NOTES

OBSERVATION

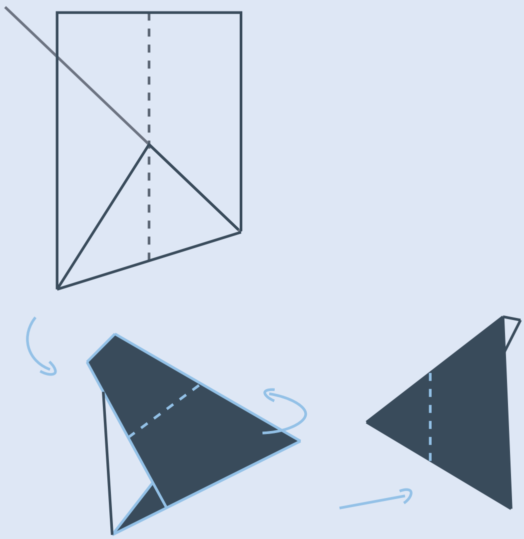
1. To fold this largest equilateral triangle from an A4 sheet, First fold the sheet in half along the length. Name this fold line as PQ.
2. Take one of the edges of the A4 sheet and put it on PQ such that the fold line passes through the other vertex.
3. Fold the remaining paper and the resulting triangle would be the largest equilateral triangle in an A4 sheet.

EXPLORE

1. Can you try to find out the side length and area of this biggest equilateral triangle.

   Hint: Cos 30° $=\sqrt{(3)}/2 = 0.87$

2. Find all the lengths and angles marked in the sheet. Assume that the sides of the rectangle are 1 and $\sqrt{2}$ units.

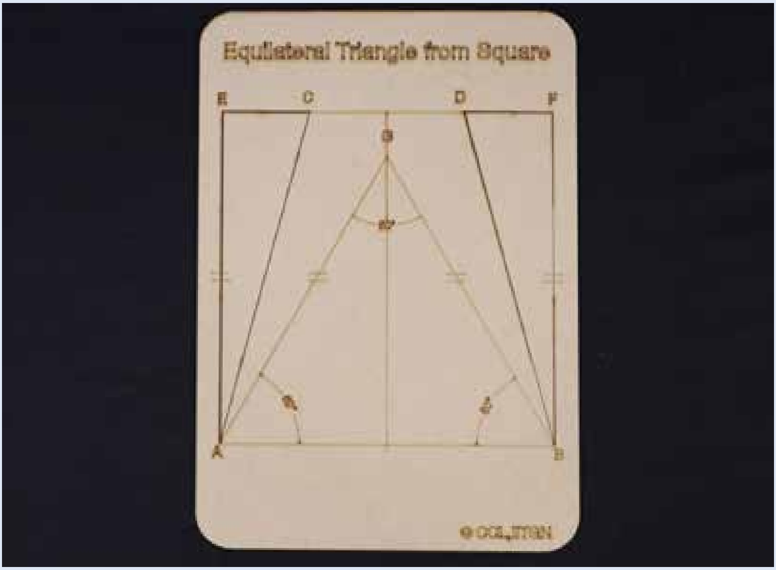


EQUILATERAL TRIANGLE FROM A SQUARE

LEARNINGS

Geometry

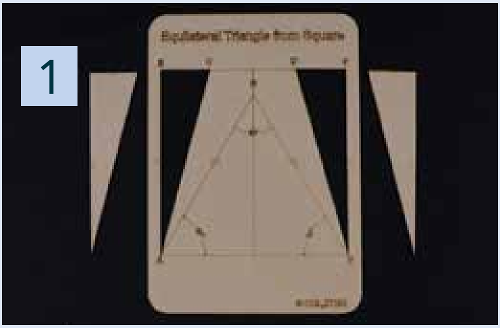
Equilateral Triangle



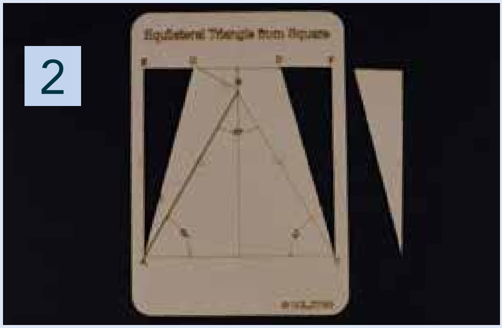
How do you convert a square into an equilateral triangle? In this activity you can convert the given square into an equilateral triangle without any measurement.

WHAT TO DO?

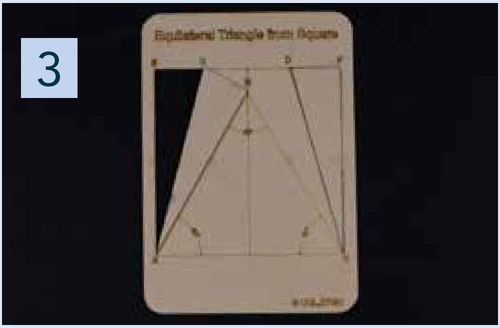
1. Put a small sticky tape in the middle of the line segments AP and BQ. Fold the triangles APD and BQE.
2. After folding, points D and E would intersect at point C which lies on perpendicular bisector of side AB.
3. ΔABC is the required triangle.



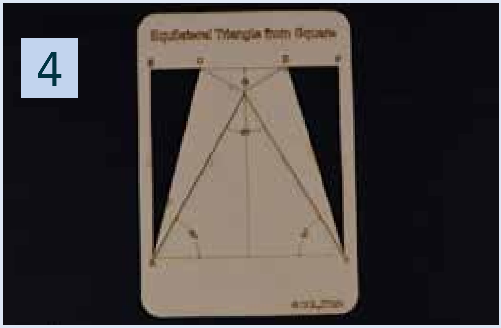
Separate the shown triangles from the sheet.



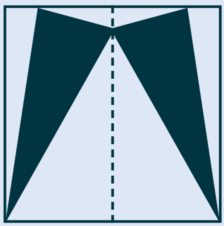
Use tape to join the triangle with segment AP.



Do the same for the triangle at segment BQ.



Fold the triangles APD and BQE.



NOTES

WHAT'S GOING ON?

1. All the four sides of a square are equal. Therefore, you can take any three sides and bring them together to form a triangle.
2. The triangle thus formed would surely be an equilateral triangle. Try this again with a square sheet of paper and fold it into an equilateral triangle!

EXPLORE

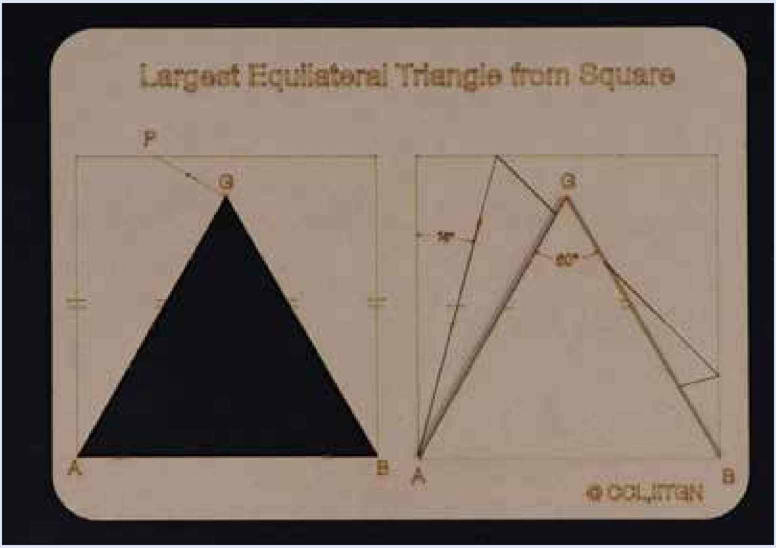
1. Find out:
   - Altitude and area of the equilateral triangle.
   - Length of DP and QE. (Hint: tan 15° = 0.27)
2. Find the other lengths and angles marked in this sheet. Assume that length of the sides of the square is 1 unit.
3. You can also cinvert the square to an equilateral triangle using the rectangle process given in the previous activity (involving perpendicular bisector)

LARGEST EQUILATERAL TRIANGLE FROM A SQUARE

LEARNINGS

Mensuration

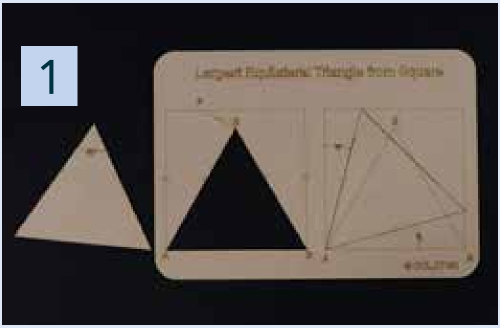
Area of a triangle
Equilateral Triangle



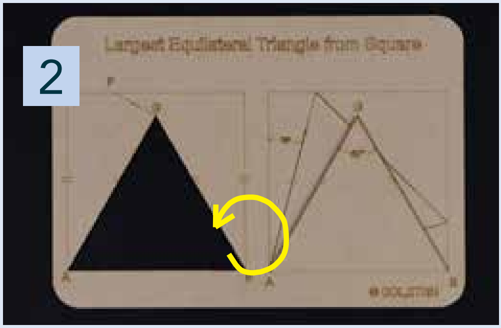
What is the largest equilateral triangle you can make from a square? Can you make an equilateral triangle whose sides are bigger than the square? This activity shows the largest equilateral triangle possible in a square.

WHAT TO DO?

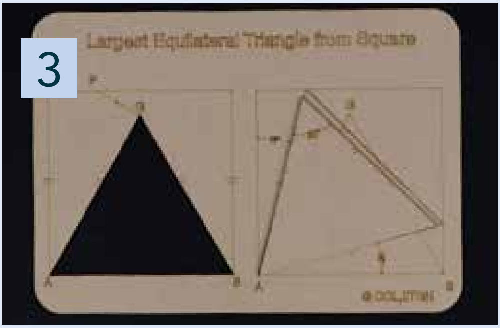
1. Take out the equilateral triangle ABC from the left square and put this along AB on the right square.
2. If you rotate this triangle about point A and extendthe sides, you can get a bigger equilateral triangle in the square.
3. $\Delta AB'C'$ is the biggest equilateral triangle you can make in the square.



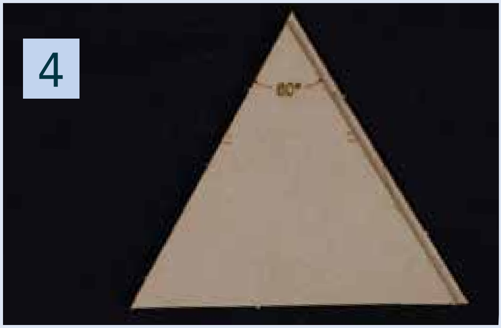
Take out the equilateral triangle ABC from the left square and put this along AB on the right square.



Rotate this triangle about point A.



On rotating ΔABC, you would see that you can get a bigger equilateral triangle in the square.



You can also compare the sizes of the two triangles.

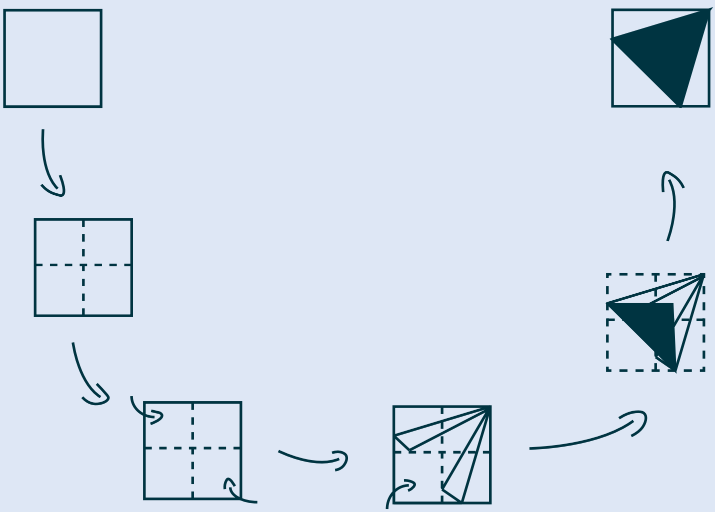
NOTES

OBSERVATION

1. When you rotate the triangle about the vertex A and extend all its sides equally, you get an equilateral triangle.
2. How much can you rotate in this manner so that the triangle still remains inside the square? It turns out to be 15°.
3. If we rotate the triangle further, vertex C goes out of the square. Therefore, ΔAB'C' is the largest equilateral triangle in this square.

EXPLORE

1. Try to find out the side length and area of this biggest equilateral triangle.
   Hint: Cos 15° = 0.97
2. Find the other lengths and angles marked in this sheet. Assume that the sides of the square are 1 unit.



GOLDEN PENTAGON

LEARNINGS

Geometry

Golden ratio

Diagonals of a Pentagon

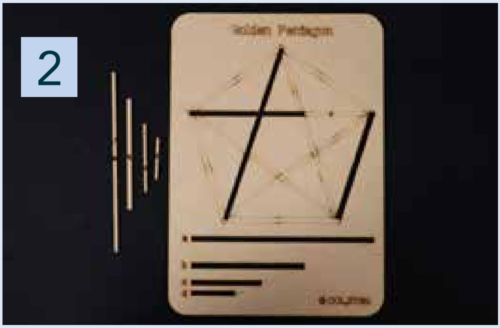
Basic Proportionality Theorem

The golden ratio makes frequent and often unexpected appearance in geometry. Regular pentagon is one of the places where the golden ratio appears in abundance. The five diagonals of a regular pentagon define a star-shaped figure called a Pentagram . It has five sides and five vertices.
In this activity, we will discuss about the various golden ratios present in a pentagram.

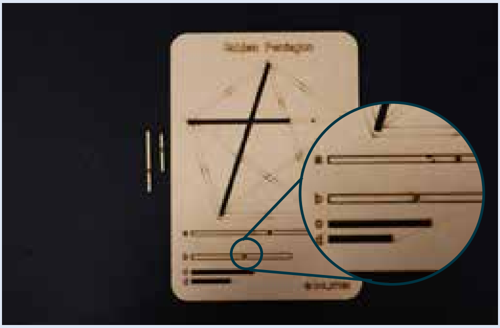
WHAT TO DO?

1. Remove the sticks marked as a, b, c and d from the sheet.
2. Measure the length a and b with the help of the scale given with the sheet and calculate the ratio a/b.
3. Similarly measure the length of c and d and calculate the ratio b/c and c/d.

Take out the strips at the bottom of the sheet and keep them aside.



Remove the sticks marked as a, b, c and d from the pentagon. Measure their lengths and calculate the ratio a/b, b/c and c/d.



Insert these sticks in the empty spaces below.

All the sticks from the pentagon would fit inside the slits below.

NOTES

OBSERVATION

1. You will find that all these three ratio are approximately equal to 1.6.
2. This ratio is called the Golden ratio, whose value is 1.618.
3. The ratio of the diagonals and sides of a pentagon is equal to golden ratio.
4. The isosceles triangles formed in the pentagon are also called golden triangles because the ratio of the length of their sides is also golden.
5. Another special property of these triangles is that they can be divided into two triangles, which are again isosceles.

WHAT'S GOING ON?

1. Suppose you have a stick and you break it such that the ratio of length of original stick to the larger part is equal to the ratio of length of larger part and smaller part. That ratio is called the Golden Ratio.
2. Can you find out that ratio using the information above? If a and b are in golden ratio (and a is the larger number), you can say, a/b = (a+b)/a. If you solve this equation, you would see that a/b is equal to 1.618.
3. By measuring the sticks a, b, c and d given in the sheet, you can see that (a+b)/a = a/b or (b+c)/b = b/c and so on.

EXPLORE

Some of the greatest mathematical minds of all ages, from Pythagoras and Euclid in ancient Greece through astronomer Johannes Kepler, to present-day scientific figures have spent endless hours over this simple ratio and its properties. But the fascination with the Golden Ratio is not confined just to mathematicians. Biologists, artists, musicians, historians, architects and even psychologists have pondered and debated the basis of its ubiquity and appeal. In fact, it is probably fair to say that the Golden Ratio has inspired thinkers of all disciplines like no other number in the history of mathematics.

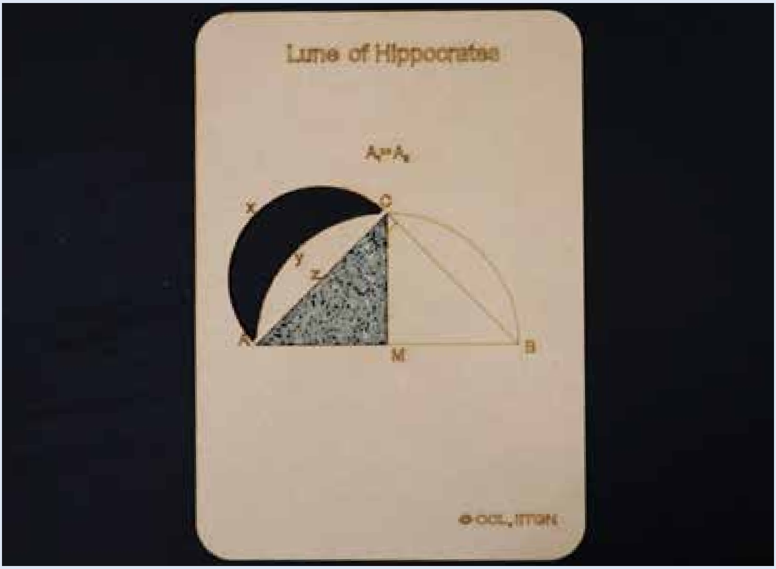
LUNE OF HIPPOCRATES

LEARNINGS

Pythagoras Theorem

Area of a Circle

Area of Right-angled Triangle

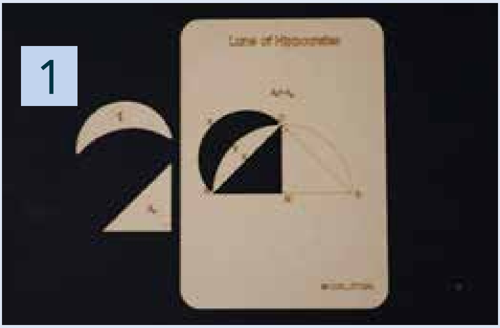


Can you make a square having same area as a given circle, without any measurements? This problem was first formulated by Hippocrates around 400 BC who wanted to square a circle.

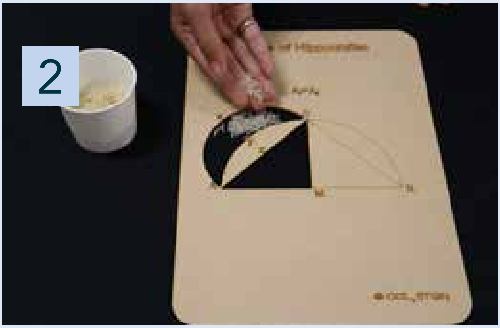
Lune is derived from the latin word Luna for moon. As this shape is surrounded by only circles, Hippocrates hoped to square the circle by solving this problem.

WHAT TO DO?

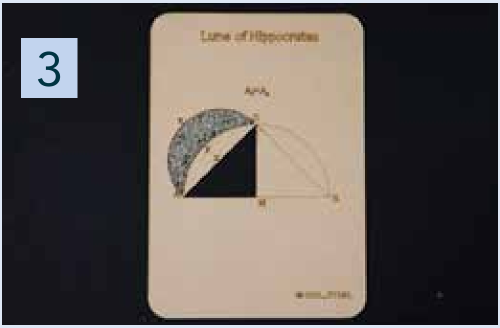
1. Take out the lune (marked as A_1) from the sheet.
2. Cover the lune completely using rice (or wheat or other small identical objects). Make sure that the seeds don't overlap with each other.
3. Take out the triangle A_2 from the sheet. Cover the area of the triangle using the same seeds.
4. You will find that the same number of seeds cover the lune as well as the triangle. This shows that both the shapes have the same area.



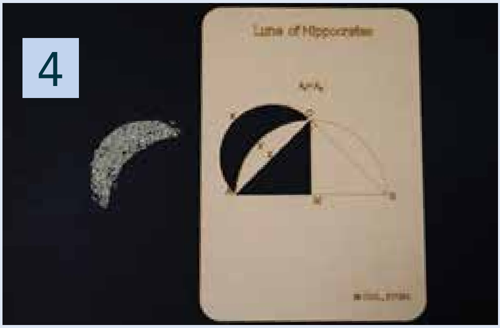
Take out the lune and the triangle (marked as A_1 and A_2) from the sheet.



Along with the sheet, you would require rice (approximately 150 g).

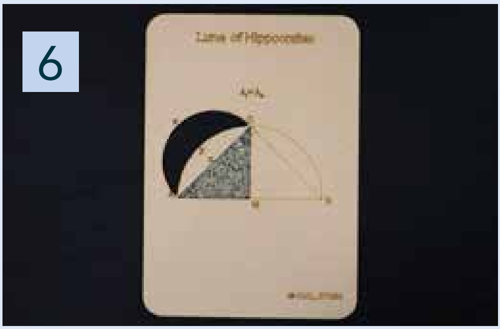


Cover the lune completely using rice. Make sure that the seeds don't overlap with each other.



Lift the sheet and shift it to the side so that the rice comes out of the lune.

Cover the area of the triangle using the same rice.



You will find that the same number of seeds cover the lune as well as the triangle.

NOTES

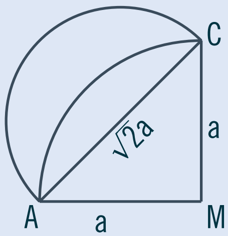
WHAT'S GOING ON?

1. Δ ACM is an isosceles right triangle (AM = CM).
2. By Pythagoras Theorem,

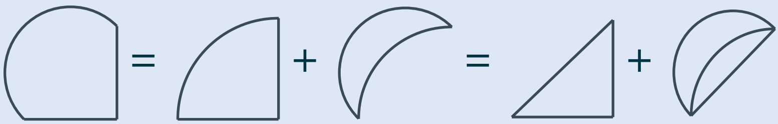
   $AC^2 = AM^2 + CM^2$

   $\quad\quad = 2AM^2$ (as AM = CM)

   Therefore, AC = $\sqrt{2}$.AM



Area of the Ice- Cream = Quarter Circle + Lune = Triangle + Semi Circle



$$\frac{1}{4} \times \pi a^2 + \text{Lune} = \text{Triangle} + \frac{1}{2} \times \left[\frac{\pi (\sqrt{2}a)^2}{4}\right]$$

$\therefore$ Area of Lune = Area of Triangle

EXPLORE

While this problem may seem routine for a geometry student today, Hippocrates was trying to solve this problem in 400 BC! At the time the Greeks did not even know the formula for the area of a circle! The Lune of Hippocrates is said to be one of the first mathematical calculations of an area between curved lines.

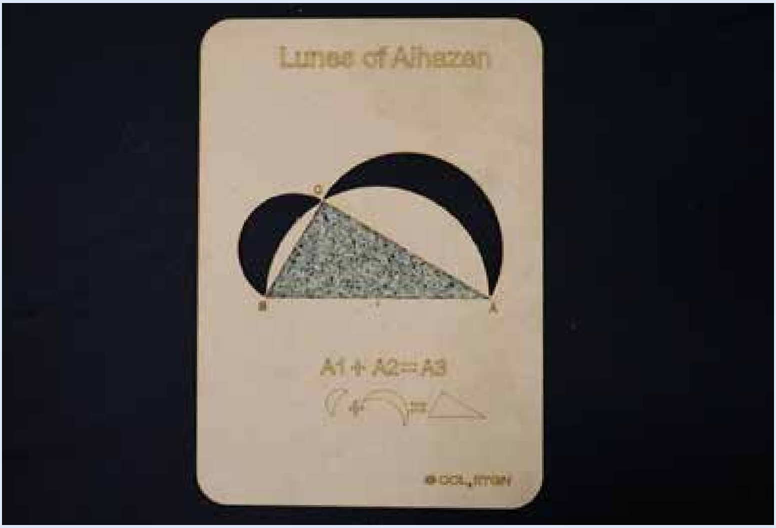
LUNES OF ALHAZEN

LEARNINGS

Pythagoras theorem

Area of a Circle

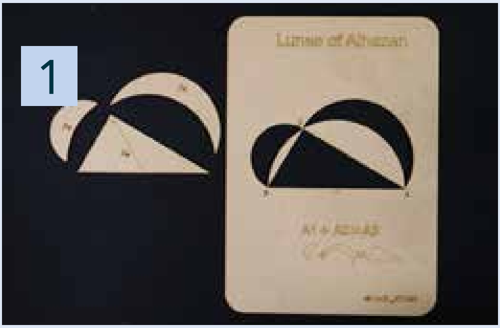
Area of Right Angle Triangle



The Arab mathematician Alhazen showed, around 1000 AD, that the areas of two lunes (moon-like shapes) added together is equal to the area of a triangle. The lunes formed this way are called Lunes of Alhazen.

WHAT TO DO?

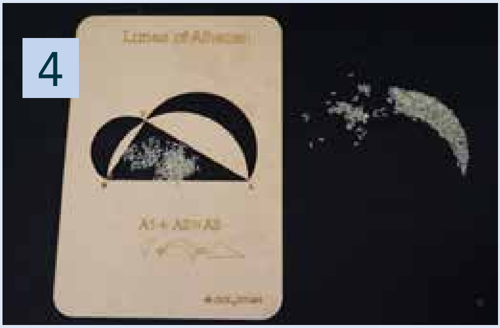
1. Take out the lunes (marked as A_1 and A_2) from the sheet.
2. Cover the lunes completely using rice (or wheat or other small identical objects). Make sure that the seeds don't overlap with each other.
3. Cover the area of the triangle (marked as A_3) using the same seeds.
4. You will find that the same number of seeds cover the lunes as well as the triangle. This shows that the total area of two lunes is equal to the area of the triangle



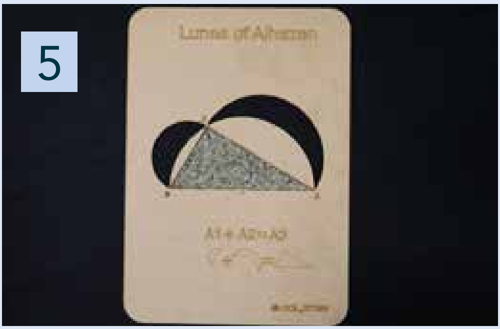
Take out the lunes (marked as A_1 and A_2) and triangle marked as A_3 from the sheet.

Cover the lunes A_1 and A_2 completely using rice.

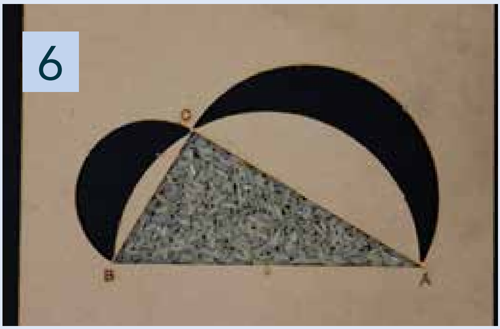
Lift the sheet and shift it to the side so that the rice comes out of the lune.



Use the same rice to fill the triangle A_3.



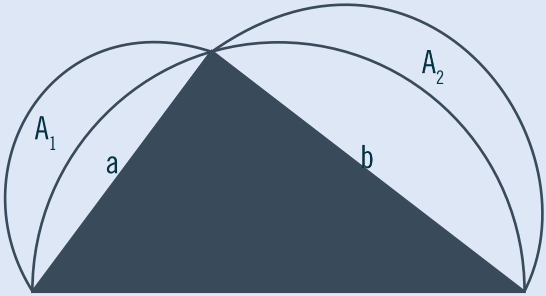
You will find that the same number of seeds cover the lunes as well as the triangle.



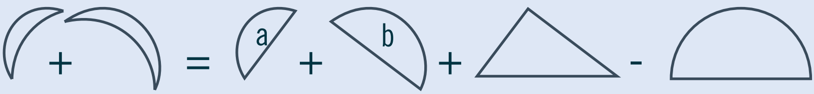
This shows that the total area of two lunes is equal to the area of the triangle

NOTES

WHAT'S GOING ON



Total Area of Lunes $= A_1 + A_2$



$$A_1 + A_2 = \frac{1}{2}\left(\frac{\pi a^2}{4}\right) + \frac{1}{2}\left(\frac{\pi b^2}{4}\right) + \triangle - \frac{1}{2}\left(\frac{\pi c^2}{4}\right)$$

$a^2 \quad + \quad b^2 \quad = \quad c^2$ [Pythagoras Theorem]

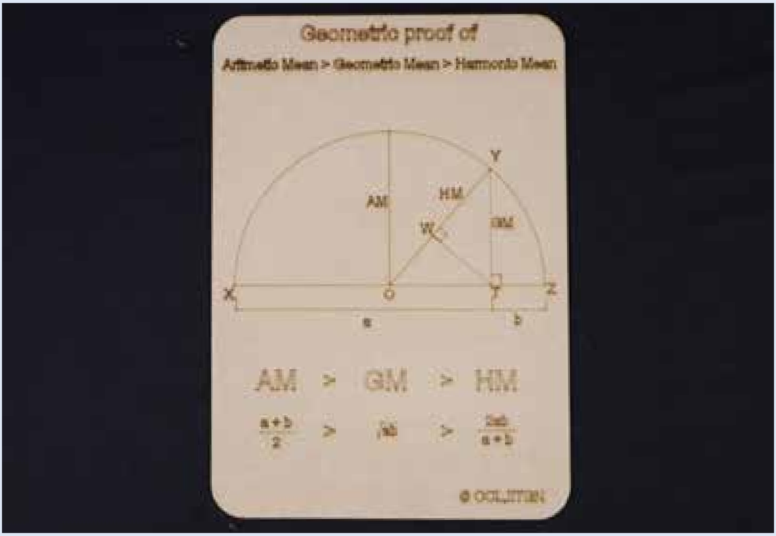
$$\therefore \quad \frac{1}{2}\left(\frac{\pi a^2}{4}\right) + \frac{1}{2}\left(\frac{\pi b^2}{4}\right) = \frac{1}{2}\left(\frac{\pi c^2}{4}\right)$$

$$\Rightarrow \quad A_1 + A_2 = \triangle$$

ARITHMETIC MEAN ≥ GEOMETRIC MEAN ≥ HARMONIC MEAN

LEARNINGS

- Algebra
- Geometry
- Pythagoras Theorem
- Similar Triangles
- Arithmetic Progression
- Geometric Progression
- Harmonic Progression



The arithmetic mean of two numbers a and b is the average of two numbers, a+b/2. Geometric mean is √ab and the harmonic mean is 2ab/a+b. For any two numbers, we have the relation between these means: Arithmetic Mean ≥ Geometric Mean ≥ Harmonic Mean

You can show this algebraic identity geometrically in this activity.

OBSERVATION

1. The two numbers a and b are marked as XT and TZ on the sheet.
2. Therefore, XZ = a + b. O is the midpoint of line XZ, so OX = OZ = OV = OY = (a+b)/2
3. Therefore **OV is the Arithmetic mean of a and b.**
4. OT = OZ - TZ = (a+b)/2 - b = (a-b)/2
5. In the right Δ OTY, using Pythagoras Theorem, $OY^2 = YT^2 + OT^2$

   Therefore, $YT^2 = OY^2 - OT^2 = [(a+b)/2]^2 - [(a-b)/2]^2 = ab$

   YT = √ab

NOTES

6. **YT is therefore the Geometric mean of a and b.**
7. WY/ YT = YT / OY (because Δ TWY and Δ OTY are similar: 90 degree and one common angle)
8. WY = YT^2/OY = $(\sqrt{ab})^2$/ [(a+b)/2] = 2ab/a+b
9. **WY is therefore the Harmonic mean of a and b.**
10. If we look at the sheet, AM≥ GM because AM is the radius of the semicircle and GM is clearly smaller than the radius.
11. GM ≥ HM because in Δ TWY, GM is the hypotenuse which is larger than other two sides.
12. Therefore, AM ≥ GM ≥HM.

LAW OF COSINES

LEARNINGS

Trigonometry

Area of a Triangle

The law of Cosines relates the length of the sides of a triangle to the cosine of its angles. But how do we arrive at the law? This sheet proves the Law of Cosines using basic trigonometry and similarity of triangles.

WHAT TO DO?

1. Draw a triangle ABC on paper as shown on the top right of the sheet.
2. Taking B as center and BC as radius, draw a circle. Extend the sides AC, BC and AB such that they intersect the circle at points E, F, G and D respectively.

OBSERVATION

1. FB = BC = BG = BD = a (all are radii of the circle).
2. AD = BD - AB = a - c
3. $\angle$FEC = 90° (angle in a semicircle is always a right angle)
4. In the right triangle FEC, Cos C = EC/FC = EC/2a

Therefore, EC = 2aCos C

NOTES

5. $AE = EC - AC = 2a\cos C - b$
6. Now, according to Intersecting Chord Theorem at point A, $AE \times AC = AG \times AD$

$(2a\cos C - b) \times b = (a+c) \times (a-c) = a^2 - c^2$

Therefore, $\cos C = (a^2 + b^2 - c^2) / 2ab$

WHAT'S GOING ON?

The Intersecting Chords Theorem can also be proved using similar triangles as follows:

$\angle GEC = \angle GDC$ (angles subtended by same chord GC)

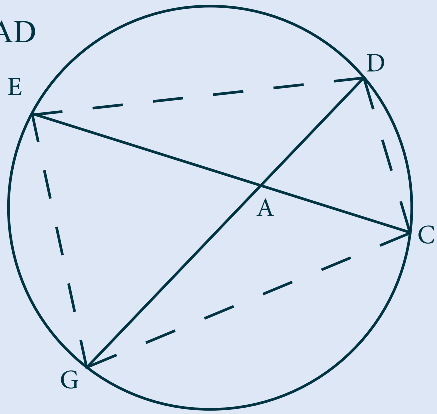
$\angle EGD = \angle ECD$ (angles subtended by same chord ED)

$\angle EAG = \angle DAC$ (opposite angles)

Therefore, ΔAEG and ΔADC are similar.

Hence, $AE/AD = AG/AC$

Therefore, $AE \times AC = AG \times AD$

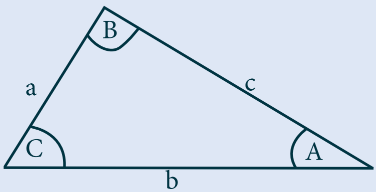


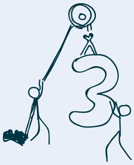
EXPLORE

The Law of Cosines can be used in conjunction with the Law of Sines to find all sides and angles of a triangle.

Law of Sines

$a/\sin A = b/\sin B = c/\sin C$



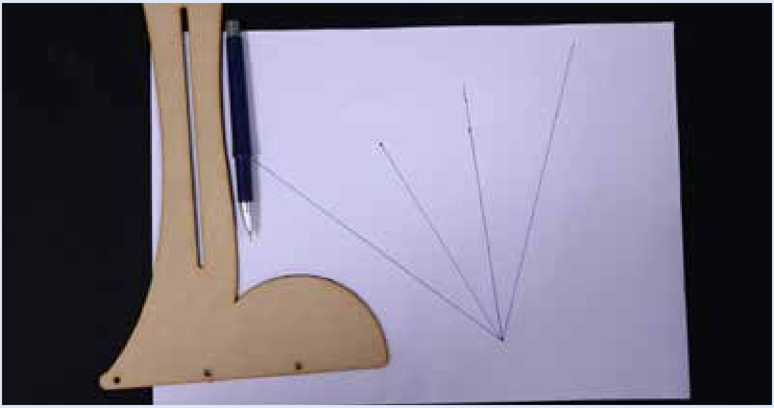


ANGLE TRISECTOR

LEARNINGS

Trisection of Angle

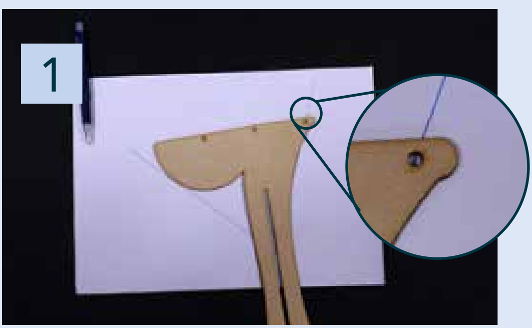
Congruency of Triangles



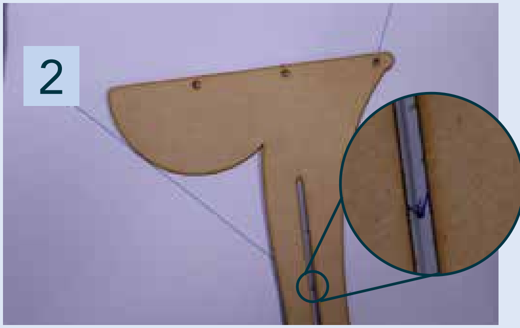
This trisector looks like a tomahawk, a Native American axe. Instead of splitting wooden logs, it is a tool for splitting an angle into three equal parts!

WHAT TO DO?

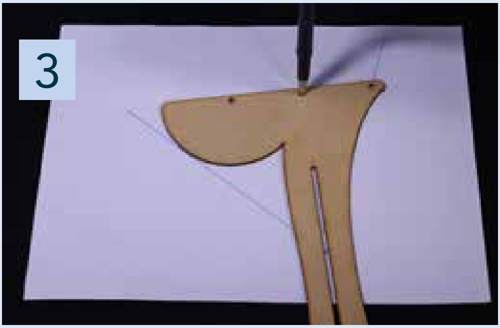
1. Take any angle you want to trisect ($\angle ABC$). The trisector has to touch this angle in following three places:
   - One ray of the angle passes through the first hole.
   - The other ray touches the semicircle tangentially
   - The angle vertex lies inside the slit.
2. With the tomahawk in this position, the angle is trisected by the handle (segment BS), and the line connecting the angle vertex to the center of the semicircle (Segment BT).
3. Do this exercise for any different angles and verify if it trisects them.



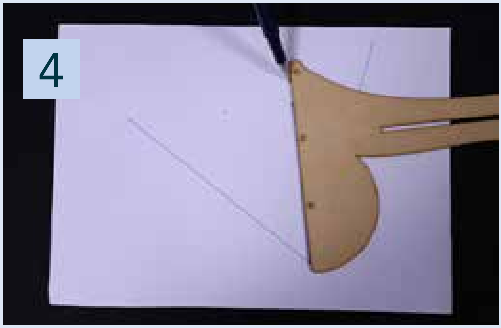
Place the trisector such that: (i) One ray of the angle passes through the first hole.



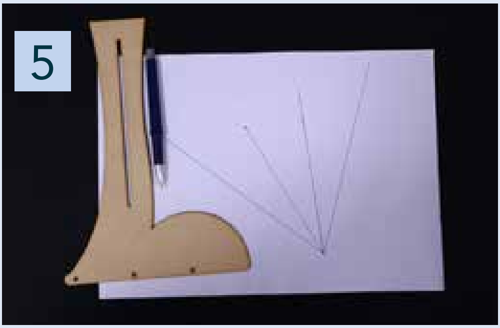
(ii)The other ray touches the semicircle tangentially
(iii)The vertex lies inside the slit.



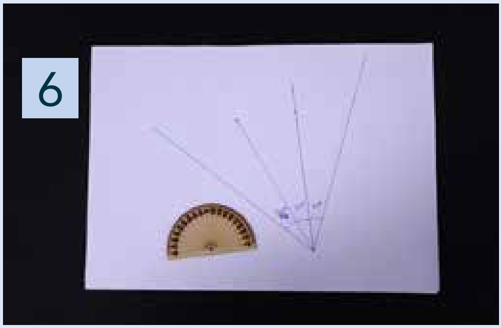
Mark the two points using pen.



Join these marked points to the vertex of the angle.



These segments divide the angle into three parts as shown.

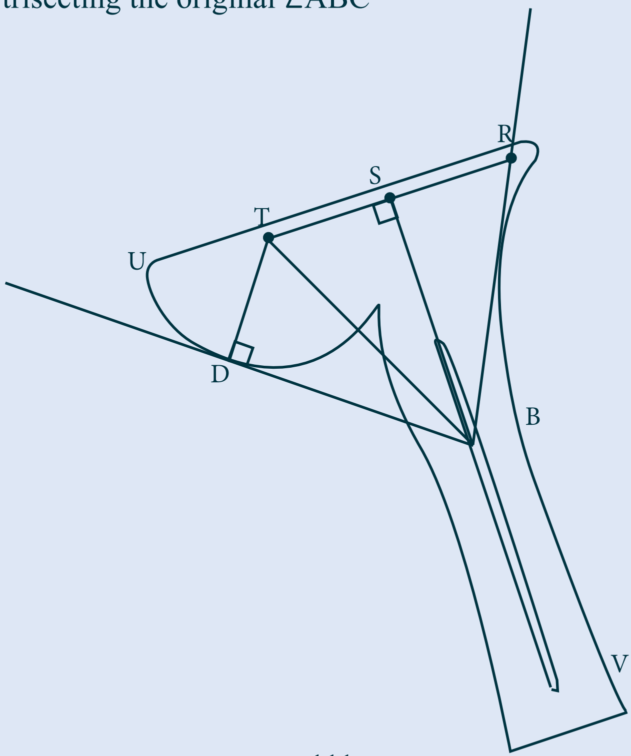


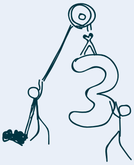
Measure the three angles and verify that they are equal.

NOTES

WHAT'S GOING ON?

1. Tomahawk consists of a segment RU, divided into three equal parts - RS, ST, and TU.
2. A semicircle with radius TS is drawn, with T as its center. SV is a line segment perpendicular to RU.
3. After placing the trisector as shown in the figure, Δ BSR and Δ BST become congruent triangles. Both triangles have right angles (∠BSR & ∠BST) with a common base (SB) and equal height (SR = ST).
4. As the sides BD and TD of Δ BDT are respectively the tangent and radius of the semicircle, they are at right angles to each other.
5. Therefor, Δ BDT and Δ BST are also congruent. Both triangles have right angles (∠BTD & ∠BST) with a common base (TB) and equal height (TD = TS).
6. This means that the three triangles are congruent to each other.
7. This means that ∠RBS, ∠SBT, and ∠TBD are equal, trisecting the original ∠ABC



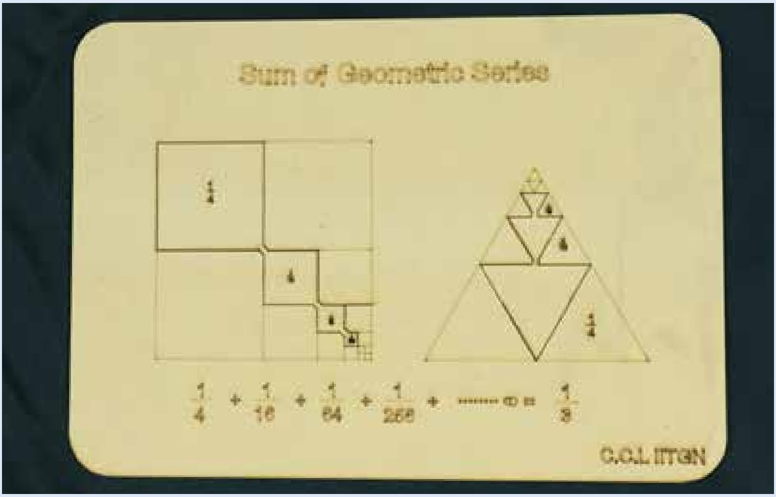


SUM OF GP (SUM = 1/3)

LEARNINGS

Geometric Progression

Area of a Square



Can you add a series that has infinite number of terms? Would the answer still be a finite number? Let's find out in this activity.

WHAT TO DO?

1. Take out the pieces marked as 1/4, 1/16, 1/64,...from the square and the triangle
2. Observe that these pieces make 1/3rd of the whole square and triangle.

NOTES

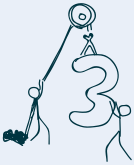
OBSERVATION

1. The side of the big square is taken as 1 unit. Therefore the area of the square is 1 square units.
2. If you divide the square in four equal parts, the area of that piece would be 1/4.
3. If you further divide that part in four equal parts, the area would be 1/4 × 1/4 = 1/16 and so on. So the area of all the three pieces is equal to 1/4 + 1/16 + 1/64 + 1/256 + ...
4. The square is divided in three equal parts by these pieces. So the area of one piece is equal to 1/3rd of the area of whole square = 1/3
5. Therefore, 1/4 + 1/16 + 1/64 + 1/256 + ... = 1/3

WHAT'S GOING ON?

The sequence 1/4, 1/16, 1/64,..... is an example of Geometric series. It's a series where ratio of two consecutive terms is same.

In this case, the ratio is 1/4 for any two consecutive terms. You can have another sequence where the difference is some other number. For example, in 1, 4, 16, 64,...(and so on), the difference between consecutive terms is 4. If this ratio is more than 1, the sum of infinite terms won't be a finite number.

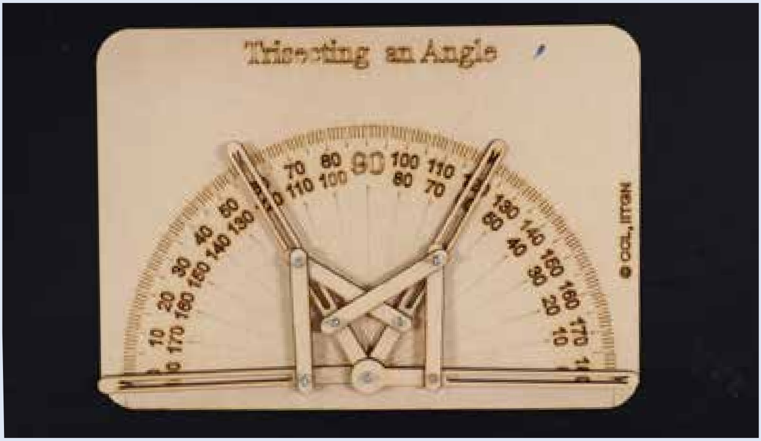


ANGLE TRISECTOR

LEARNINGS

Trisection of Angle

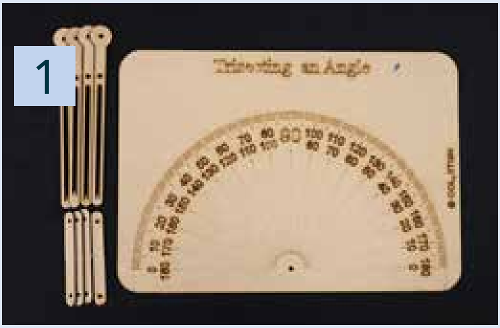
Congruency of Triangles



Angle trisection is the division of an angle into three equal parts. The mechanism in this module automatically trisects any angle and is called Laisant's mechanism. Explore the geometry in the mechanism.

WHAT TO DO?

1. Connect the four arms and connecting rods as shown in the images using nuts and bolts.
2. Connect the mechanism at the hole in the center of the sheet.
3. Put the first arm at the zero angle and the last arm at 180°. The two middle arms would point to 60° and 120°, dividing the angle 180° into three equal parts.
4. Move the first and last arm at any position and you would observe that the middle arms would automatically trisect the angle.

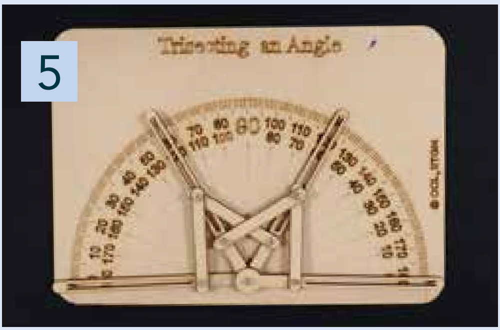


You will require the sheet, nuts and bolts.

Use a nut and bolt to attach the two parts as shown.

Attach the second arm and the connecting rod.

Attach the remaining parts. The final structure would look like this.



Fix the arms to the middle of the sheet with the help of a screw.

NOTES

WHAT'S GOING ON?

1. This mechanism for trisecting an agnle is called the Laisant's Compass, proposed by M. Laisant in 1875.
2. The lengths are chosen so that OB = OC = OD = OA and CS' = BS' = AS = DS, with S and S' as joints permitted to slide in straight grooves along the two trisecting bars.
3. Therefore, ΔOBS', ΔODS and ΔOAS are congruent. Hence, ∠BOS' = ∠DOS = ∠SOA, dividing ∠BOA in three equal parts.